



**UNIVERSITY OF THE PHILIPPINES BAGUIO**

**Bachelor of Science in Mathematics**

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**Announcement of the Bachelor's Thesis Examination**

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for the degree of

**BACHELOR OF SCIENCE IN MATHEMATICS**

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This is to certify that the Undergraduate Thesis entitled ”**Insert Title of Your Thesis,**” prepared and submitted by **Juan Cruz** and **Juan Cruz**, to fulfill part of the requirements for the degree of **Bachelor of Science in Mathematics**, was successfully defended and approved on July 2024.

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This Undergraduate Thesis is hereby officially accepted as partial fulfillment of the requirements for the degree of Bachelor of Science in Mathematics.

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# Abstract

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This is the content of the Abstract.

# Acknowledgments

Thank you.

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# Chapter 1

## Introduction

Cells of the sinoatrial (SA) node of the heart fire impulses in synchrony to help the atria contract. [1, 5].

# Chapter 2

## Preliminaries

A discourse on the background of the study is in this chapter. We divided this chapter into three parts.

### 2.1 Basic Concepts on Synchronization

We will use mainly the definitions from the book of Hoppensteadt and Izhikevich [3], and Pikovsky et al. [4] and adapt some terms in the current literature.

**Definition 2.1.1 (Phase Synchronization (PS))** A set of coupled oscillators is said to achieve *phase synchronization* (or *synchronized in phase*) if there exists a time  $t_0$  such that

$$|\theta_m(t) - \theta_n(t)| = 0, \quad n, m \in \mathcal{V}, \quad \text{for all } t \geq t_0,$$

# Chapter 3

## Dynamic Theory of Modified Models

Our results focus on the long-term dynamics of the systems.

### 3.1 On Modified Kuramoto Model

In this section, we consider the following Kuramoto Model

$$\dot{\theta}_n(t) = \omega_n + \frac{\kappa}{N} \sum_{m=1}^N \sin(\theta_m(t) - \theta_n(t)). \quad (3.1)$$

# Chapter 4

## Numerical Simulations

In the course of the discussions of our analytical results.

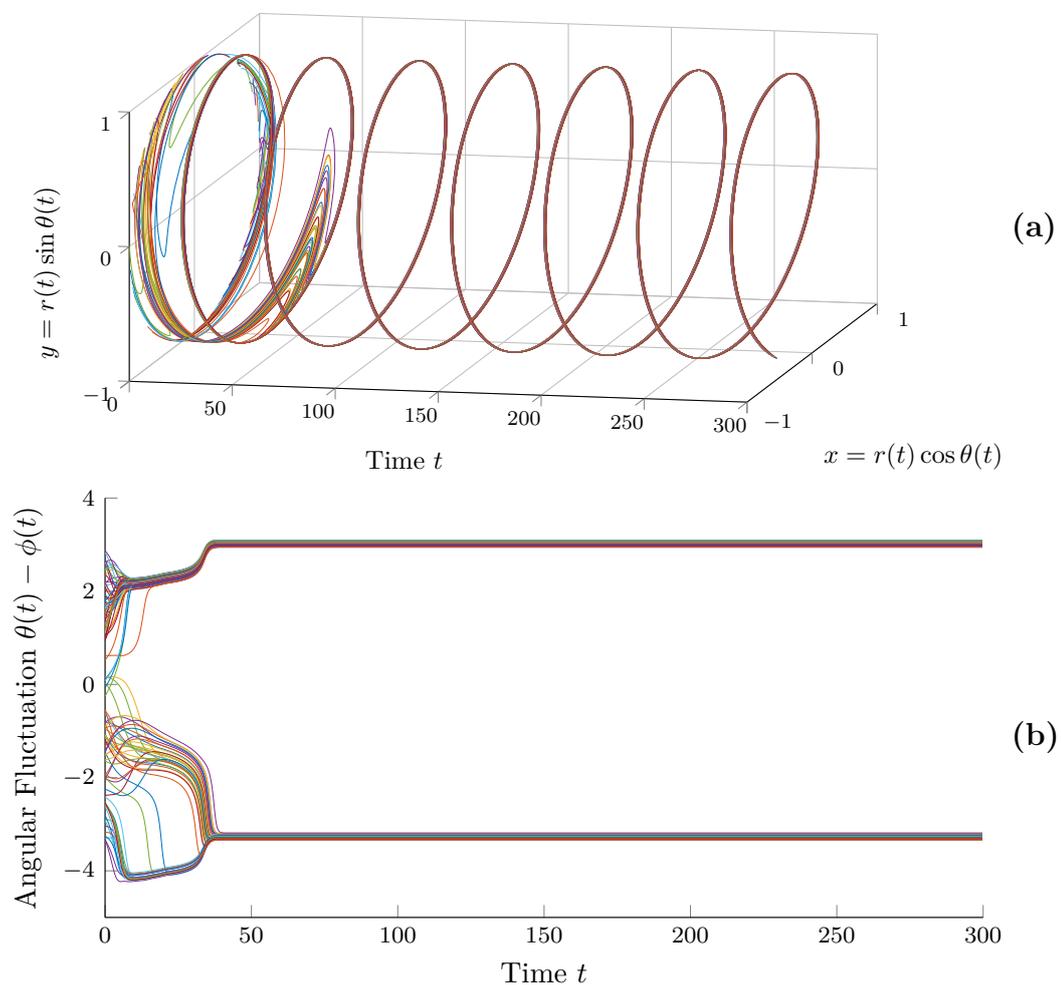
### 4.1 Asymptotic Frequency Synchronization

We see here that the approximation error is less than  $10^{-11}$ .

**Table 4.1.** The table above shows the maximum values of  $g$

$r$	$\gamma_{\text{exp}}$	$g(\gamma_{\text{exp}})$	$M(\gamma_{\text{exp}})$	$r$	$\gamma_{\text{rat}}$	$g(\gamma_{\text{rat}})$	$M(\gamma_{\text{rat}})$
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

We can see from the following values of  $r$  in Table 4.1. that  $M(\gamma) < g(\gamma)$  for coupling function  $f$ .



**Figure 4.1.** The above figures show Asymptotic Frequency Synchronization of modified Kuramoto Model with exponential coupling function.

# Chapter 5

## Conclusions and Future Work

### 5.1 Summary and Conclusions

Conclusion

### 5.2 Recommendations for Future Works

Future Works

# Appendix A

## Notations

A.1 Basic

A.2 Functions

# Appendix B

## Notes on Other Theorems

We extend their result by considering a system.

**Remark B.0.1** *The theorem in [2] follows by taking  $\hat{\mathbf{r}} = (1, \dots, 1)^T$  in the previous theorem.*



## List of References

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