

DELFT UNIVERSITY OF TECHNOLOGY

AE3212-II SIMULATION, VERIFICATION & VALIDATION  
(2015-2016 Q3)

GROUP A03

---

# Fokker F-100 Simulation Plan

---

*Authors:*

Casper DEK, 4300653

Kostas FINES, 4223934

Jesse EPPINK, 4290429

Juan Esteban MONTOYA, 1545167

Tim Rudolf PAULSEN, 4225589

Matthijs REEKERS, 4233905

February 15, 2016



# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Group Organization</b>	<b>2</b>
2.1	Workpackage Composition . . . . .	2
2.2	Gantt Chart . . . . .	3
2.3	Task Allocation . . . . .	4
<b>3</b>	<b>Problem Analysis</b>	<b>5</b>
3.1	Inputs and outputs . . . . .	5
3.2	General Assumptions . . . . .	5
3.2.1	Primary Assumptions . . . . .	5
3.2.2	Secondary Assumptions . . . . .	6
3.3	Free body diagrams and reference axis system . . . . .	6
<b>4</b>	<b>Proposed Analytical Solution</b>	<b>8</b>
4.1	Bending . . . . .	8
4.2	Shear Stress . . . . .	9
<b>5</b>	<b>Proposed Numerical Solution</b>	<b>10</b>
5.1	Assumptions . . . . .	10
5.1.1	Primary Assumptions . . . . .	10
5.1.2	Secondary Assumptions . . . . .	11
5.2	Governing Equations . . . . .	11
5.2.1	Geometric Properties . . . . .	11
5.2.2	Internal Forces and Moments . . . . .	11
5.2.3	Internal Stresses . . . . .	12
5.3	Numerical Method . . . . .	12
5.3.1	Geometric Properties Code Block . . . . .	13
5.3.2	Internal Forces and Moments Code Block . . . . .	13
5.3.3	Internal Stresses Code Block . . . . .	13
<b>6</b>	<b>Verification Strategy</b>	<b>15</b>
6.1	Code Verification . . . . .	15
6.2	Calculation Verification . . . . .	15
6.2.1	Unit Tests . . . . .	16
6.2.2	Addressing Discrepancies . . . . .	16
6.3	System Test . . . . .	17
<b>7</b>	<b>Validation Strategy</b>	<b>17</b>
7.1	Validation Experiment . . . . .	18
7.2	Measurement Selection . . . . .	18
7.3	Addressing Discrepancies . . . . .	18

## List of symbols

Symbol	Description	Unit
$\epsilon$	Horizontal distance to shear center	mm
$\eta$	Vertical distance to shear center	mm
$\eta_s$	Distance from shear center to centroid	mm
$\theta$	Rotation	rad
$\frac{d\theta}{dz}$	Rate of twist	$\frac{rad}{mm}$
$\sigma_i$	Normal stress in discretized section	MPa
$\sigma_{z1}$	Normal stress caused by bending moment around x-axis	MPa
$\sigma_{z2}$	Normal stress caused by bending moment around y-axis	MPa
$\tau_{xy}$	Shear stress in the x-y-plane	MPa
$b$	Distance between booms	mm
$d_{lg,y}$	Distance between bottom of the fuselage and bottom of the wheels	mm
$d_{tail,z}$	Distance tail aerodynamic center to aft of the fuselage	mm
$g$	Gravitational acceleration	$\frac{N}{s^2}$
$v$	Deflection	rad
$q$	Distributed deceleration force	$\frac{N}{m^2}$
$q_s$	Shear flow	$\frac{N}{m}$
$q_{s0}$	Constant shear flow caused by torque	$\frac{N}{m}$
$q_{s0,i}$	Constant shear flow in discretized section caused by torque	$\frac{N}{m}$
$q_b$	Basic shear flow caused by shear	$\frac{N}{m}$
$q_{b,i}$	Basic shear flow in discretized section caused by shear	$\frac{N}{m}$
$t$	Skin thickness	mm
$t_i$	Skin thickness discretized section	mm
$t_D$	Idealized skin thickness	mm
$A$	Area	mm <sup>2</sup>
$A_{stringer}$	Stringer area	mm <sup>2</sup>
$A_i$	Discretized area	mm <sup>2</sup>
$B_i$	Boom area	mm <sup>2</sup>
$F_T$	Horizontal tail force	N
$G$	Modulus of rigidity	MPa
$I_{xx}$	Area moment of inertia around x-axis	N
$I_{xy}$	Product moment of inertia around	N
$I_{yy}$	Area moment of inertia around y-axis	N
$L_x$	Horizontal reaction force of the left main landing gear	N
$L_{f1}$	Distance from front of the fuselage to the nose landing gear	mm
$L_{f2}$	Distance from the nose- to the main landing gear	mm
$L_{f3}$	Distance between the left- and right main landing gear	mm
$L_y$	Vertical reaction force of the left main landing gear	N
$M_x$	Bending moment around x-axis	N
$M_y$	Bending moment around y-axis	N
$N_x$	Horizontal reaction force of the nose landing gear	N
$N_y$	Vertical reaction force of the nose landing gear	N
$S_x$	Horizontal shear force	N
$S_y$	Vertical shear force	N
$R_x$	Horizontal reaction force of the right main landing gear	N
$R_y$	Vertical reaction force of the right main landing gear	N
$T$	Torque	Nmm
$X_{analytical}$	Numerical output variable	—
$X_{numerical}$	Numerical output variable	—

# 1 Introduction

This simulation plan aims to explain and document all steps to be taken in the simulation analysis of the Fokker-100 case. This case is the scenario in which the aircraft touches the ground during landing, introducing loads on the landing gear. The landing gear consists of 2 rear gears around mid-span and one in the front. Furthermore, the pilot uses the rudder which induces a lateral force on the tail.

As stated in the assignment, it is required to find the maximum normal stress and location of this in the fuselage, the maximum shear stress and its location in the fuselage, and the maximum shear flow in the frames that introduce the loads from the landing gears into the fuselage.

From these requirements, the problem statement which the simulation will be built for, is defined as:

”Given the data of the Fokker-100 and the landing scenario, what are the maximum shear and normal stresses at their critical locations in the fuselage and the maximum shear flow in the frames?”

The work done will give several output variables that can be compared to experimental data and are listed below.

- Maximum normal stress in the fuselage and its location
- Maximum shear stress in the fuselage and its location
- Maximum shear flow in the in the frames

This simulation plan consists of 6 parts, of which the first is the group organization in section 2. In the group organization section the work is divided in parts, the time necessary to finish it is estimated and divided among the group members. Secondly, the problem analysis will be done in section 3. This part consists of the governing equations, the general assumptions and the reference frame. The third section is on the proposed analytical solution and can be found in section 4. Its counterpart, the proposed numerical solution, can be found in section 5. After the solutions a proposal for the Verification and Validation and will be given, which can be found in sections 6 and 7 respectively.

## 2 Group Organization

In this section, the organization of the group over the period of the project will be discussed. Section 2.1 presents the composition of the workpackages, section 2.2 presents the Gantt chart task distributions, and 2.3 presents the task allocation amongst the team members.

### 2.1 Workpackage Composition

This project has been split up into 4 workpackages, which are derived from the 4 phases of this project. The workpackages, with their respective goals are presented as follows.

#### WP1 - Initial phase

1. Research problem and project outline.
2. Generate problem statement and output variables.
3. Project planning for next phases.
4. Analyse the problem and determine the governing equations.
5. Generate analytical and numerical solution approaches.

6. Define verification and validation strategies.
7. Generate and hand in simulation plan.

### WP2 - Simulation phase

1. Define conceptual, mathematical and computational models.
2. Determine analytical solution.
3. Determine and program numerical solution.

### WP3 - Verification & Validation Phase

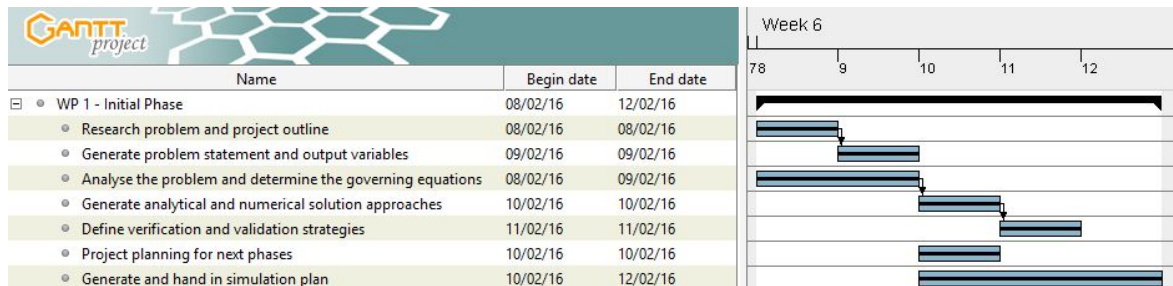
1. Perform code and calculation verification.
2. Define and perform validation experiments.
3. Perform uncertainty assessment.

### WP4 - Reporting phase

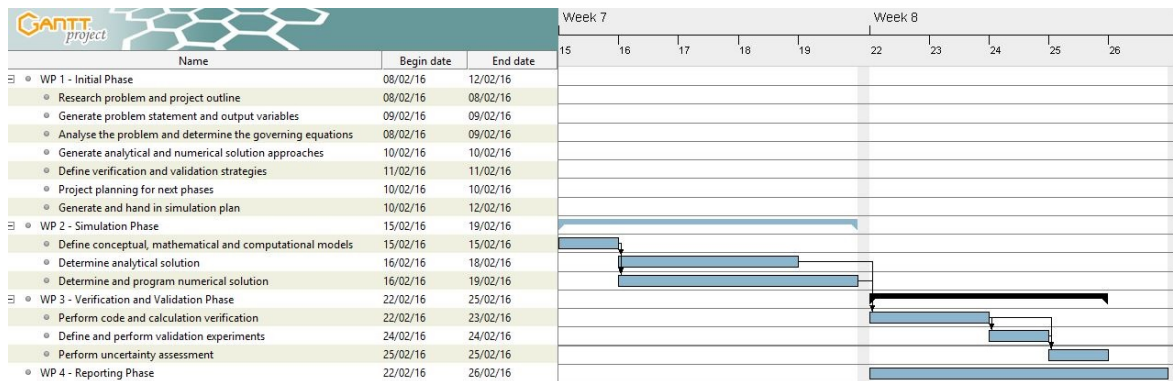
1. Document all work done and outcomes in the final report.

## 2.2 Gantt Chart

A Gantt chart has been created, which will be used to monitor the progress of this project. This was created using a third party software [1]. The tasks in the Gantt chart are derived from the previously mentioned workpackages. This Gantt chart can be found in figure 1. It is important to note that the solid black line in the middle of all tasks in the WP 1 tasks workpackages.



(a) Week 1



(b) Week 2 and 3

Figure 1: Gantt chart of Work Packages

## 2.3 Task Allocation

The tasks are allocated as shown in table 1. It is important to note that the estimated time is the approximated time which is required to complete the task. This is the estimated time per person. Deadlines are defined at 12.30 in the afternoon of that particular date.

Table 1: Task allocation per team member

Task	Responsible	Estimated Time (hours)	Other participating members	Deadline
WP 1		29		
1	Kostas	4	Casper, Jesse, Tim, Juan, Matthijs	09-02-2016
2	Casper	3	-	09-02-2016
3	Kostas	4	Matthijs	12-02-2016
4	Tim	5	Casper, Jesse, Matthijs	10-02-2016
5	Jesse	5	Casper, Kostas, Tim, Juan, Matthijs	10-02-2016
6	Kostas	4	Juan	11-02-2016
7	Matthijs	4	Casper, Jesse, Tim, Juan, Kostas	12-02-2016
WP 2		20		
1	Jesse	5	Casper, Kostas, Tim, Juan, Matthijs	17-02-2016
2	Casper	5	Jesse	22-02-2016
3	Tim	10	Juan, Kostas, Matthijs	22-02-2016
WP 3		18		
1	Juan	5	Kostas, Matthijs, Tim, Jesse	24-02-2016
2	Kostas	4	Casper, Juan	24-02-2016
3	Matthijs	4	Casper	25-02-2016
WP 4		5		
1	Matthijs	5	Casper, Jesse, Tim, Juan, Kostas	26-02-2016

### 3 Problem Analysis

This section presents the analysis of the problem at hand. Section 3.1 presents the inputs and outputs of the problem. Section 3.2 presents the general assumptions being used. Finally, section 3.3 elaborates on the free body diagrams and static equilibrium equations concerning the case.

#### 3.1 Inputs and outputs

In this section, the input and output variables of this structural problem are discussed. The input variables are categorized in external forces, reaction forces, fuselage dimensions and material properties. They are listed in table 2.

Table 2: Problem inputs

Property	Symbol	Value (Unit)
Length of the fuselage	$L$	28.0 (m)
Length parameter	$L_{f1}$	2.0 (m)
Length parameter	$L_{f2}$	14.01 (m)
Length parameter	$L_{f3}$	5.04 (m)
Fuselage radius	$R$	1.6 (m)
Floor height	$f_h$	1.2 (m)
Skin thickness	$t_s$	2.0 (mm)
Floor thickness	$t_f$	1.5 (cm)
Thickness of stiffener	$t_{st}$	1.2 (mm)
Height of stiffener	$h_{st}$	1.5 (cm)
Width of stiffener	$w_{st}$	2.0 (cm)
Number of stiffeners (equally spaced along the periphery of the cross-section)	$s_n$	36 (-)
z-distance of aerodynamic centre of the tail to the back of the fuselage tail	$d_{tail,z}$	1.8 (m)
y-distance of aerodynamic centre of the tail to the back of the fuselage tail	$d_{tail,y}$	4.1 (m)
y-distance between bottom of the fuselage and the landing gear	$d_{lg,y}$	1.2 (m)
Lateral force on the tail	$S_x$	$8.58 \times 10^4$ (N)
Design landing mass	$W$	35.000 (kg)

The outputs of this problem are listed below.

- Normal stress in z-direction  $\sigma_z$
- Shear stress in xy-plane  $\tau_{xy}$
- Critical stress locations
- Maximum shear flow in the frames  $q$

#### 3.2 General Assumptions

The assumptions made during this project are divided in primary assumptions and secondary assumptions. Primary assumptions are have a major impact on the results and secondary assumptions have a minor impact on the results, but are still not negligible [2]. The following primary and secondary general assumptions, except number six, are given from the problem statement [3].

##### 3.2.1 Primary Assumptions

1. The total weight (structure + payload) can be modeled as a uniformly distributed load  $q$ . This has impact on the moment around the x-axis, since in the real situation a

large of the weight will be situated at the wing. Therefore the locations of maximum stress will change location.

2. The fuselage can be modeled as a beam, which has no holes due to windows. As a result of this, the outcome of the simulation will give higher maximum stresses than actually possible. In reality, the moment of inertia will be smaller at some places where there are cut-outs in the structure for windows. Furthermore, stress concentrations at the edges of these holes may also cause failure.
3. The reaction loads at the location of the landing gear can be modeled as point loads.
4. Neglect drag and assume that during landing the lift forces are negligible. Drag results in a normal force in the fuselage. This normal force compresses the fuselage, lowering the stress at locations under tension and increasing the stress at locations under compression.
5. The dynamic landing problem is assumed to be a static load case. Loads are actually applied as a function of time and the responses in terms of displacements and stresses are also varying in time. Occurrences as fatigue stress are neglected.

### 3.2.2 Secondary Assumptions

1. Landing gear provides roller support, with  $F_z = 0$ , and  $F_x, F_y$  nonzero.
2. The attachment of the floor and the stiffeners to the fuselage do not have to be analyzed.
3. The fuselage cross-section can be assumed to be constant throughout the length, and the y-axis is an axis of symmetry.
4. Vertical deceleration during “touch-down” is  $3g$ .
5. Frames can be modeled as very stiff open “rings” and are loaded in pure shear only.
6. There are no initial imperfections or eccentricities present in the structure. Imperfections or eccentricities can positively or negatively influence the strength of the structure when there is more or less material added than prescribed, or the structure is coincidentally made in a shape that handles the stresses better or worse. Furthermore, cracks could be present due to previous load cases, reducing the strength of the structure, meaning that the actual structure might be less strong than the model with which calculations are being done.

### 3.3 Free body diagrams and reference axis system

Free body diagrams are obtained for the x-z-, y-z- and x-y-planes. They can be found in figures 2, 3 and 4. For clarity, the reference frames are shown outside of the structure. When performing either the analytical or numerical solution, the origin (0,0,0) will be located at the aft of the fuselage in the centroid of the cross-section. The vertical reaction forces  $R_y$  and  $L_y$  caused by the main wheels of the landing gear act through the same longitudinal(z)- location. These reaction forces are visible in figure 3. It should also be noted that the magnitudes of the reaction forces have not yet been determined, therefore the shown sizes and directions are arbitrary. The axis system used is the Cartesian coordinate system (x,y,z) illustrated in this section. Polar coordinates are used briefly when determining stringer coordinates in the cross section and curing certain calculations if convenient.



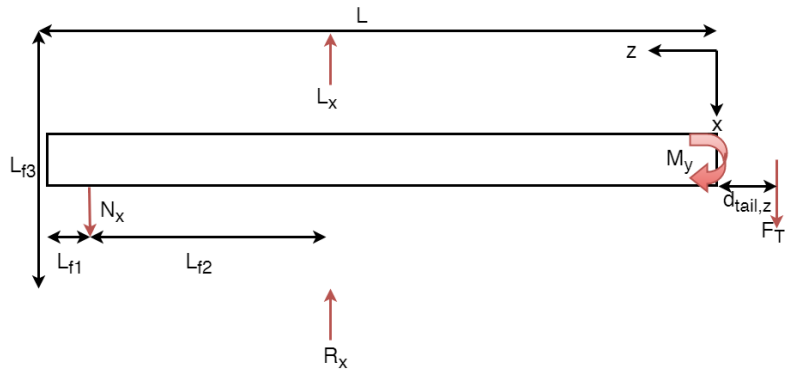


Figure 2: Free body diagram of the x-z-plane

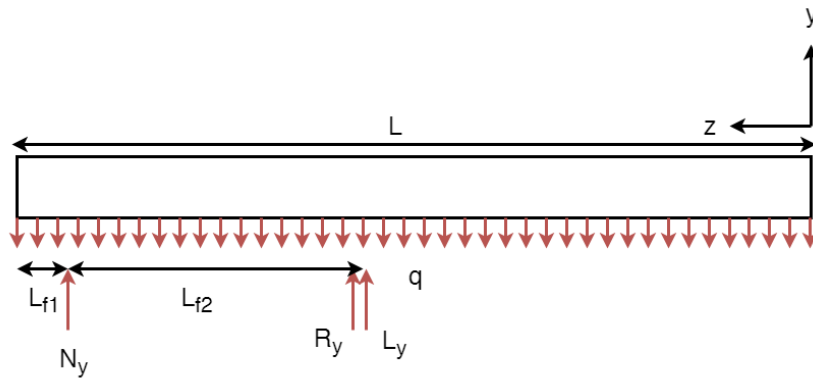


Figure 3: Free body diagram of the y-z-plane

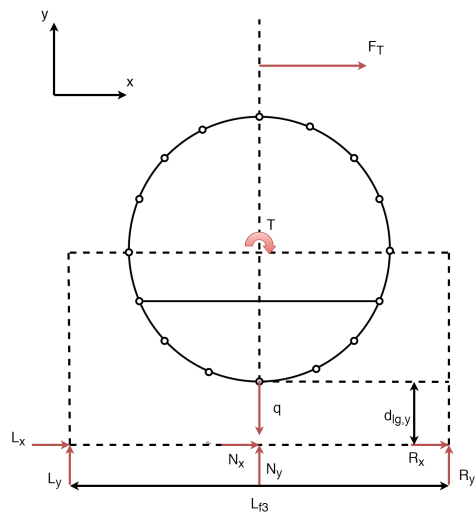


Figure 4: Free body diagram of the x-y-plane

From these free body diagrams, static equilibrium equations can be obtained. There are 6 unknown reaction forces;  $N_x$ ,  $N_y$ ,  $R_x$ ,  $R_y$ ,  $L_x$  and  $L_y$ . The static equilibrium assumption only provides 5 useful equations due to the fact that there are no forces in z-direction.

1.  $\rightarrow^+ \sum F_x = 0$
2.  $\uparrow^+ \sum F_y = 0$
3.  $\sum M_x = 0$
4.  $\sum M_y = 0$
5.  $\sum M_z = 0$

Because there are more unknowns than equations, this problem is statically indeterminate [4]. Energy methods can be used to solve this statically indeterminate problem [5].

## 4 Proposed Analytical Solution

There are 3 output variables to be determined. The first one, the maximum normal stress and its location, will be elaborated on in section 4.1. Then, the steps to come to the maximum shear stress and flow and its locations will be provided in section 4.2.

### 4.1 Bending

The beam will be analyzed in two different planes, namely the z-y and the z-x plane. Since there are no forces in z direction are present, these will not be mentioned in both planes. First, the z-y plane is considered. The bending will be caused by the distributed weight and the reaction forces of the landing gears. Now, the bending moment as a function of z can be determined by taking the moment around z=0. With this bending moment  $M_x(z)$  around the x-axis the normal stress in the structure can be determined using equation 1.

$$\sigma_{z_1} = -\frac{M_x(z) \cdot y}{I} \quad (1)$$

In which y is the distance from the centroid to the location where the normal stress is determined.

In the z-x plane the bending moment is caused by the lateral shear force of the rudder and the reaction forces on the landing gear in x direction. The same approach as for the z-y plane is used, with a similar equation 2.

$$\sigma_{z_2} = -\frac{M_y(z) \cdot x}{I} \quad (2)$$

In which x is the horizontal distance from the centroid to the location where the normal stress is determined and  $M_y(z)$  is the moment around the y-axis.

Now, to combine those two they are both expressed in polar coordinates and then added up. The resulting formula is a function of z and  $\Phi$  and can be found in equation 3.

$$\sigma_{tot}(z, \Phi) = -\frac{M_x(z) \cdot R \cdot \sin(\Phi)}{I} - \frac{M_y(z) \cdot R \cdot \cos(\Phi)}{I} \quad (3)$$

This formula now has to be differentiated with respect to z and  $\Phi$  to find the coordinates and value of the maximum normal stress.

## 4.2 Shear Stress

When predicting the consequences of the shear stress on the structure, it has to be mentioned that there is both shear applied in the x-direction, due to the force on the tail and the landing gear as well as in the y-direction, due to the distributed force  $q$  and the normal forces provided via the wheelbases.

To determine if the horizontal shear forces also causes a torque, the shear center has to be known. If the shear passes through the shear center, no extra torque will be present. This is the case for the shear force in the y-direction, since it passes through the center and the shape is symmetric in the y-plane. This is not the case for the x-direction, resulting in torque. The approach for the calculation of the consequences of the shear, is to apply the shear in the shear center, whilst adding an additional torque.

The shear center can be determined by applying an arbitrary horizontal shear force at a distance  $\eta_s$  from the centroid of the cross section. The centroid of the cross section can be found using equation 4. The torque around the centroid created by this shear force should be equal to the torque created by all shear flows in the cross section. The corresponding value for  $\eta_s$  is the distance of the shear center from the centroid. The torque created by the shear flows can be determined by cutting the cross section into different parts and calculate the created torque for every section. For curved sections, this is done using equation 5 [6] and for straight sections this is done using equation 6 [6], multiplied with distance  $d$  from the centroid.

$$\bar{y} = \frac{\Sigma A \cdot y}{\Sigma A} \quad (4)$$

$$M_q = \int_0^s q(s) \cdot p ds \quad (5)$$

$$F_q = \int_0^s q(s) \cdot ds \quad (6)$$

Now the cross section can be split up between all the booms, which gives curved sections, and the straight horizontal floor. By adding all these separate torques and make them equal to the torque created by the applied shear force the vertical distance of the shear center from the centroid is found. Since the cross section is symmetrical around the y-axis, the shear center will also be on this axis. Therefore, the location of the shear center is known when the coordinate on the y-axis is determined.

In order to solve for the shear load, the problem is considered to be a two-cell beam. The skins will not be idealized, while the stiffeners will be considered as booms. First, the cells will be cut and  $q_b$  will be calculated. For both cells, the shear flows are determined by equation 7.

$$q_b = -\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \left( \int_0^s t_D x ds + \sum_{i=1}^n B_i x_i \right) - \frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \left( \int_0^s t_D y ds + \sum_{i=1}^n B_i y_i \right) \quad (7)$$

The rate of twist for the two cells is assumed as equal. Thus, in order to solve, the twist according to equation 8 will be set equal for both cells.

$$\frac{d\theta}{dz} = \frac{1}{2A} \oint \frac{q_s}{tG} ds \quad (8)$$

Now one more equation is needed to solve for the shear stress, since  $q_{s0}$  is not yet determined. This is obtained by moment equivalence in the form of equation 9. The moment caused by the applied loads in the two directions about any point must be equal to the moment caused by the resulting shear flows about that point.

$$S_x \eta + S_y \varepsilon = \int_0^s p q_b ds + 2A q_{s0} \quad (9)$$

The total shear flow is now determined by adding all shear flows in equation 10.

$$q_s = q_b + q_{s0} \quad (10)$$

With this set of equations, the shear flow at all locations in the structure can be calculated. Since the cross-section is the same at all locations in the z-direction, the locations that will be examined will be determined by the shear diagram that will be constructed.

In order to get the actual shear stress, the shear flow simply has to be divided by the skin thickness  $t$ , as stated in formula 11.

$$\tau = \frac{q}{t} \quad (11)$$

## 5 Proposed Numerical Solution

The numerical analysis will be carried out with a software tool developed in Python. The software tool will analyze a discretized representation of the fuselage consisting of a number of sections along the z-axis and 36 evenly spaced points coincident with the cross section in the x-y-plane. In this chapter, the specific assumptions concerning the numerical analysis are listed along with the equations and proposed general structure of the software tool. All theory is based on [5]. Section 5.1 presents assumptions that are used in this method. Section 5.2 presents the governing equations. Section 5.3 presents the numerical method.

### 5.1 Assumptions

The specific assumptions that will be used for the numerical analysis are listed below and categorized into primary and secondary. Primary assumptions have a relatively large impact on the results compared to the secondary assumptions, although the effect of these cannot be neglected either.

#### 5.1.1 Primary Assumptions

1. Boom idealization: Stringers and skin are lumped together in what is referred as booms and consists of two main simplifications. Due to the cross section of the stringers being open and having relatively small dimensions compared to the rest of the fuselage, their shear stress is neglected and neutral axis is assumed coincident with that of the skin. Furthermore the normal stress is assumed constant along the cross section of the stringers. The skin is assumed to not carry any normal forces. The area of the skin is therefore distributed to the location of the stringers and their areas lumped together. Skin and stringers are now represented as booms. Rather than cross section shape, the booms and their locations are now determine stress resistance and might either over- or underestimate the final values depending on the original geometry and how it is idealized.
2. Equal twist and torque equivalence: Both cells of the cross section are assumed to experience equal twist, meaning the shape of the cross section remains unchanged. Torque equivalence means the total torque is equal the sum of torques experienced by each cell. This simplifies the problem by allowing the cross section to be considered a rigid body.

### 5.1.2 Secondary Assumptions

1. Thin-wall: The thickness of the skin, stringers and floor are small relative to the fuselage as a whole, therefore the contribution of terms with higher orders of  $t$  will be neglected when calculating moments of inertia. In addition the thickness is ignored in defining distances. This will lead to a slight overestimation of normal and shear stresses due to area moment of inertia in general being lower.

## 5.2 Governing Equations

A number of equations will be utilized by the software tool. These equations can be sorted into three different areas of interest: Geometric Properties, Internal forces and moments and Internal stresses, each with their accompanying code block in the software tool.

### 5.2.1 Geometric Properties

The cross section is symmetrical about the y-axis, meaning it is the vertical neutral axis and that  $I_{xy}$  is zero. The horizontal neutral axis position is determined with equation 12. Area moment of inertia is determined using equations 13 and 14. Equation 15 is used with skin idealization and in simplifying the stringers.

$$\bar{y} = \frac{\int_A \tilde{y} dA}{\int_A dA} \quad (12)$$

$$I_{xx} = \int_A y^2 dA \quad (13)$$

$$I_{yy} = \int_A x^2 dA \quad (14)$$

$$B_i = A_{stringer} + \frac{t_D b}{6} \left( 2 + \frac{\sigma_{i+1}}{\sigma_i} \right) + \frac{t_D b}{6} \left( 2 + \frac{\sigma_{i-1}}{\sigma_i} \right) \quad (15)$$

### 5.2.2 Internal Forces and Moments

The equations for internal shear forces in x- and y-directions and moment around the x- and y-axis are determined by force equilibrium around some cut made along the length of the fuselage and will all be functions of  $z$ . The fuselage is in this case defined by three main areas as described in subsection ???. What external forces are taken into account for the internal shear equations 16 and 17, and moment equations 18 and 19, will depend in which area the cut was made.  $z$  in this case denotes the moment arm and  $q$  in equation 17 denotes the distributed force representing the weight.

$$S_x(z) = \sum F_{x,i} \quad (16)$$

$$S_y(z) = \int_0^z q dz + \sum F_{y,i} \quad (17)$$

$$M_x(z) = \iint q dz + \sum F_{y,i} z_i \quad (18)$$

$$M_y(z) = \sum F_{x,i} z_i \quad (19)$$

### 5.2.3 Internal Stresses

For each of the discretized sections the stresses of interest are the normal stress in z-direction throughout the whole cross section along with shear stress in the x-y-plane. The normal stress will be determined throughout the fuselage using equation 20 at all boom locations in every fuselage section, keeping in mind that  $I_{xy}$  is zero due to symmetry.

$$\sigma_z = \frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} + I_{xy}^2} y + \frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} + I_{xy}^2} x \quad (20)$$

In order to determine the shear stress throughout the fuselage, the total shear flow will be defined over the multicell cross section of each fuselage section. Shear flows due to shear forces,  $q_b$ , will be determined by making a cut at some convenient location and using equation 21, keeping in mind that  $I_{xy}$  is zero,  $B_i$  represents boom areas and skin thickness,  $t_D$ , is assumed zero (except for the floor).

$$q_b = -\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \left( \int_0^s t_D x ds + \sum_{i=1}^n B_i x_i \right) - \frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \left( \int_0^s t_D y ds + \sum_{i=1}^n B_i y_i \right) \quad (21)$$

The constant shear flow of each cross section cell is determined through the use of equation 22, 23 and 24, based on the assumptions of torque equivalence, equal rate of twist between the two cells and moment equivalence respectively. Sum of torques in equation 22 is due to potentially having to translate more than one shear force and the sum on the right hand side is for one constant shear flow per cell in the cross section.

$$\sum T_i = \sum 2A_i q_{s0,i} \quad (22)$$

$$\left( \frac{d\theta}{dz} \right) = \frac{1}{2A_i G} \oint \frac{q_{b,i} + q_{s0,i}}{t_i} ds = 0 \quad (23)$$

$$S_x \eta + S_y \varepsilon = \int_0^s p q_b ds + 2A q_{s0} \quad (24)$$

The shear stress at any point is determined by dividing the total shear flow at that point with the skin thickness at that same point using equation 25.

$$\tau_{xy} = \frac{q}{t} \quad (25)$$

## 5.3 Numerical Method

Figure 5 provides an overview of the functionality of the software tool illustrating three main code blocks and input-output variables of each calculation element. Internal force, moment and stress calculation blocks (blue and red) form the loop that runs along the z-axis from the back to the front of the fuselage. The number of n sections determines the number of iterations and has a major influence on accuracy as the continuous shear force and moment distributions along the length of the fuselage are considered as discrete values. Larger numbers of n leads to a result closer to reality. It should be noted that most element outputs are arrays containing values for one cross section in the x-y-plane.

If the cross section of the current iteration was the forward most, all the results are presented with the largest values highlighted. If it was not the forward most section a step forward is made in z to the next and the calculations repeated.

### **5.3.1 Geometric Properties Code Block**

The cross section of each individual section will be discretized into 36 points, each with a set of  $x$  and  $y$  coordinates, coinciding with the given location of the stringers. This goes hand in hand with boom theory where the stringers and skin will be lumped together at these locations. The floor is either discretized into  $n$  number of sections along its width or considered a uniform bar depending on the problem at hand. These are then gathered in an array which can then be duplicated  $n$  times due to the geometry of the fuselage being constant along its length. The resulting array then consists of the  $x$ ,  $y$ , and  $z$  coordinates of all discrete points of the fuselage.

### **5.3.2 Internal Forces and Moments Code Block**

As mentioned in subsection 5.2.2, which external forces contribute to the internal shear force and moment equations depend on in which of the three areas the current fuselage section is located. The shear forces and moments are determined as scalars for a certain value of  $z$  as they are the same throughout the  $x$ - $y$ -plane of a cross section.

### **5.3.3 Internal Stresses Code Block**

Normal stress and shear flows are calculated at all the  $x$ - $y$ -coordinates, coinciding with the boom locations, of the currently analyzed cross section and through the floor using the equations mentioned in subsection 5.2.3. When all fuselage sections are analyzed the frame shear flow is determined along with normal and shear stress values. These can then be further processed or searched through to determine the maximum occurring stresses.

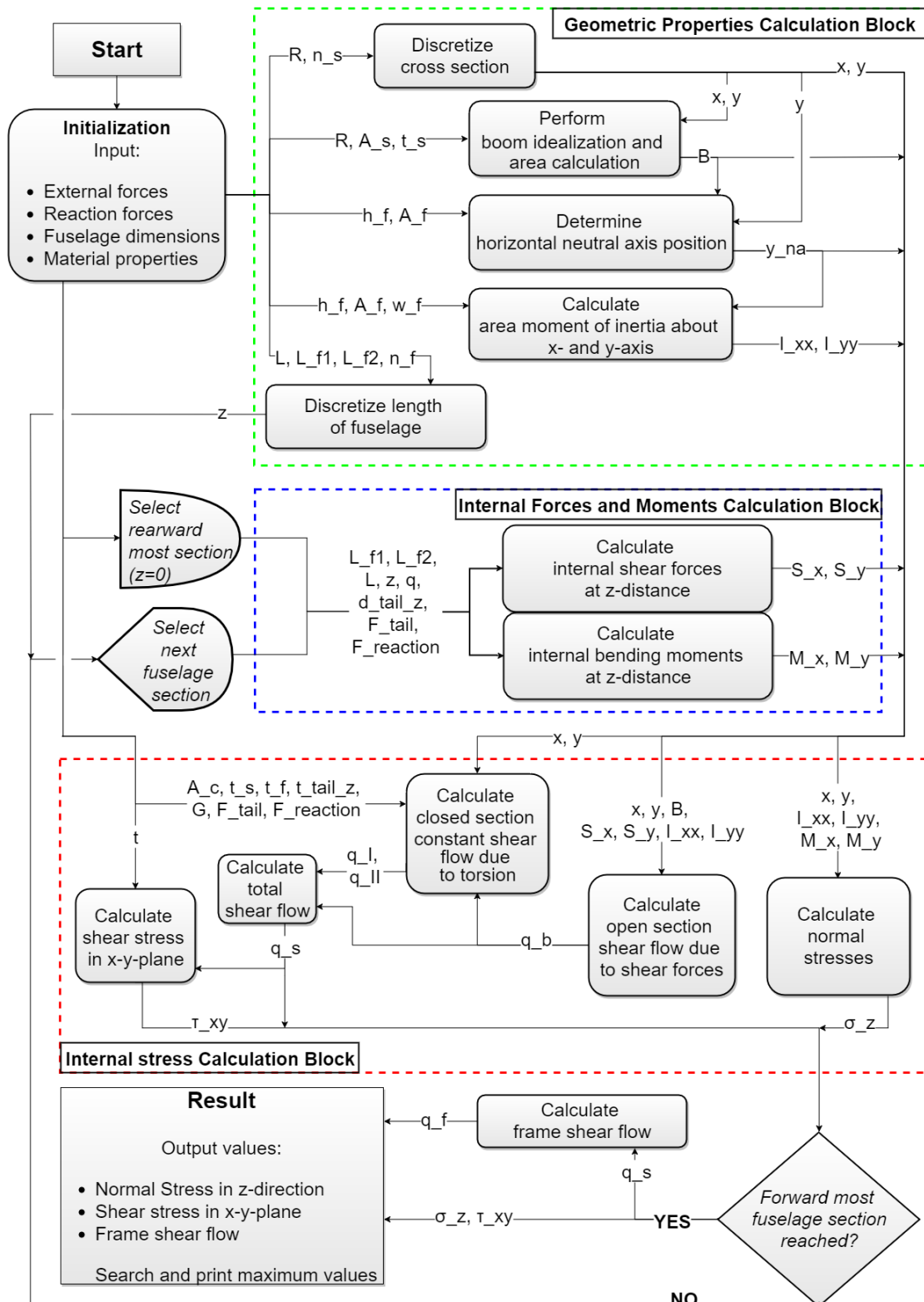


Figure 5: Software flow diagram illustrating the proposed method of stress analysis of the fuselage in three main blocks where the blue and red blocks are part of a loop.



## 6 Verification Strategy

Verification is the process of determining how accurately the numerical model represents the mathematical model and its solution.[2] In the case of this project, it is defined as the process of determining the accuracy of the created numerical solution program. It is efficient to perform verification on the whole program, including intermediate program blocks (unit tests), such that any errors can be found and fixed promptly. First, code verification will be undertaken on the whole program, and all code blocks. This is discussed in section 6.1. After all issues here have been solved, calculation verification will be undertaken, as discussed in section 6.2. Theory is taken from examples and information presented in the lecture notes[2].

### 6.1 Code Verification

Code verification is the process of finding and fixing errors that are present in the coding of the numerical solution program.[2] This is the first step of verification, where all blocks and lines of code will be examined carefully for mistakes. This section will explain how this is done.

Initially, the compiler alerts of any syntax errors. Therefore, these alerts must be addressed and fixed. Some may be trivial to fix, others may not.

After these compiler addressed issues are fixed, mistakes that make the program function differently than intended (or use extra computing power) will be addressed. Each mathematical equation that is programmed must be checked for typos, or any other mistakes. Additionally, there may be blocks of code that are redundant, or "hang" the program up by being inefficient, or having a mistake (such as a for loop that never stops).

The reference frame setup used during the implementation of the equations must also be checked to match that of the system setup, as discussed in section 3. For example, there might be incorrectly typed "-" numbers or variables.

Finally, the code should also be checked for consistency and accuracy in the values. Values that need to be floated should be confirmed as being floated, and values that should not should also be checked whether they are not. This will eliminate any "coding" errors in the calculation verification, due to numerical inaccuracy.

### 6.2 Calculation Verification

Calculation verification, which follows code verification, is the process of checking whether or not the numerical model is correct.[2]

The output value of the numerical solution will, most likely, not exactly match the theory, or analytical solution, since it is a model. An important factor that will determine the difference between the numerical model and the analytical solution is the percentage error, calculated in equation 26.

$$\%error = \frac{|X_{numerical} - X_{analytical}|}{|X_{analytical}|} \cdot 100\% \quad (26)$$

Where  $X_{numerical}$  and  $X_{analytical}$  are the outputs of the numerical and analytical solution respectively. There are many simplifications and assumptions made in the modeling of this problem. It is decided that the preliminary allowable % error will be between 10-20%, based on engineering judgment and as mentioned as an example in the lecture notes[2]. If the error is greater than this, then a discrepancy exists. The strategy used to determine why the discrepancy exists will be discussed in section 6.2.2.

### 6.2.1 Unit Tests

Intermediate verification (unit tests) of code blocks will be performed on each of the code blocks, since there may be blocks of code that function perfectly, and return incorrect outputs due to an error. This incorrect output might, in the final output of the program, cancel out or not be detectable, thus it is important that this check is performed for each individual code block.

This will be done by using common inputs in both the numerical code block and analytical method, calculating the outputs of both (analytical will be done by hand, numerical by inputting in the code block), and comparing these outputs using the % error check mentioned in section 6.2. These inputs will cover a range of points, and include trivial inputs such as 0, as well as cases that might create singularities. This enables verification over a large number of points. In the program being created, an example of unit test verification will be performed as follows:

1. Geometric Properties Code Block: Choose a discretized cross section. Calculate and compare the numerical and analytical solutions of its centroid  $x$  and  $y$ , boom idealization area  $B$ , neutral axis  $y_{na}$  and moment of inertias  $I_{xx}$  and  $I_{yy}$ . The output values of both methods should fall within the allowable % error, otherwise a discrepancy exists. A further check that could be performed (to see if discretization is correctly functioning) is to make a 3d plot of the  $x$  and  $y$  coordinates of the points of the discretized fuselage cross section, at each discretized fuselage length location  $z$ , to see if it looks like the fuselage.
2. Internal Forces and Moments Code Block: Any input values of any discretized cross section can be chosen. Using these, calculate and compare the numerical and analytical solutions of the internal shear forces  $S_x$ ,  $S_y$  and the bending moments  $M_x$ ,  $M_y$ , acting on it. The output values of both methods should fall within the allowable % error, otherwise a discrepancy exists.
3. Internal Stress Code Block: For any discretized fuselage section, calculate and compare the numerical and analytical solution of the normal stress  $\sigma_z$ , shear stress  $\tau_{xy}$  and the frame shear flows  $q_f$  for the given inputs of the discretized fuselage section. For example, taking a shear force value of 0 is expected to result in a shear flow of 0 in the cross section. The output values of both methods should fall within the allowable % error, otherwise a discrepancy exists.
4. Maximum Values of Results Code Block: this is a simple test where any random values can be inputted, and the maximum values should be outputted.

If discrepancies are present, then the items and steps stated in section 6.2.2 will be applied and checked.

### 6.2.2 Addressing Discrepancies

If there are any discrepancies identified from the unit tests in section 6.2.1, then the following checks will be performed for the code block which has the discrepancy, in order to determine why it exists and to try and improve the % error:

- Check for typos or incorrect application of formulae from theory. For example, the block might have been modeled with respect to the wrong reference plane, or the wrong equation might have been entered. Additionally, ensure that code verification, as stated in section 6.1 is re-performed.
- Perform a dimensional analysis. Compare what the dimensions of the numerical and of the analytical solution method are. They are expected to be the same. If not, there is a problem which should be addressed.

- As can be seen, there are also some intermediate variables that are not outputs, but exist in the Internal Stress Calculation Block, namely  $q_s$ ,  $q_I$ ,  $q_{II}$ ,  $q_b$ . They should be verified with their analytical solution to determine whether the internal workings of the block function correctly.
- Check the assumptions of the numerical method. An investigation should be carried out to check whether all assumptions are valid, are not over simplifying to the problem, are implemented correctly in the code block and are appropriately justified by theory. For example, boom idealization may be over simplifying to the problem, and cannot be neglected, since the difference between it's contribution to the numerical solution is too big with respect to the analytical solution.
- Check the assumptions of the analytical method. They might be over simplifying in a similar way as discussed in the previous bullet point, for example.
- Check the governing equations. This should be done for both the numerical and analytical method. There may be some mistakes here.
- Check the program architecture. There may be incorrect variable links, or issues in the way the input/outputs are handled. Additionally, the fuselage may be split into too few discretized cross sections, which means that the numerical solution is simply not accurate enough (there are not enough values) to approach the analytical solution.

If the above points have been performed and the discrepancy still exists, it could be determined that the assumptions used in the numerical or analytical model do not allow an accuracy of the allowable % difference, as stated in 6.2. Thus, the preliminary allowable % error should be revised as necessary. However, this should only be considered as a very last resort.

### 6.3 System Test

After unit tests have been performed and all discrepancies have been addressed and solved, the whole system can be tested. The program is executed for the given input variables, at many locations along the fuselage and cross sections. The analytical solutions at these locations, using the same input variables are also calculated. The final numerical solution outputs ( $\sigma_{z,max}$ ,  $\tau_{xy,max}$  and  $q_{f,max}$ ) are compared to the analytical results of these. If there are no discrepancies (using the allowable % error, as stated in section 6.2), the system, and therefore program, can be considered as verified and the validation procedure may be commenced.

If there are discrepancies, then there is a problem somewhere. Code and calculation verification should be re-performed, paying even closer attention. If there are no coding errors, then the issue will most likely lie in the assumptions of the numerical and analytical methods, as stated in section 6.2.2.

## 7 Validation Strategy

Validation is the determination of the predictive ability of the numerical, computational model.[2] This phase occurs after verification. Successful validation is accomplished when the predictive capability of a computational model fulfills the required accuracy set beforehand. This predictive capability is determined by comparing the simulation outputs with the experimental outputs. Since finances and time-management are important priority drivers in every real-life engineering project, it is important to validate with an appropriate strategy. Section 7.1 discusses the validation experiment, section 7.2 discusses the measurement selection, and section 7.3 discusses how discrepancies should be addressed. Theory is taken from examples and information presented in the lecture notes[2].

## 7.1 Validation Experiment

A validation experiment is important for generating experimental data with which the computational model can be compared to. It is important to note that the experimental setup, initial conditions, boundary conditions and input parameters must be defined correctly and as accurately as possible. However, in this project, the experimental data will be given. Therefore, the validation strategy will focus on how this will be used to validate the numerical program.

## 7.2 Measurement Selection

The model's intended use is to serve as the guideline for selecting the quantities to measure during the experiment. The experimental data that is measured should be, ideally, in the same units, planes, directions, locations and coordinate system as the model simulates. If this is not the case, then the experimental data will be converted such that it matches the outputs of the computational model. In the case of this project, the maximum stresses with their respective location, and the maximum shear flow of the frames are the experimental values of interest.

The two data sets must then be compared at all (or as many) of the locations of the fuselage and conditions as possible. As previously stated, the % error described in equation 26 will now serve as a useful tool at seeing the difference between the numerical and experimental outputs. The smaller this error, the more "valid" and therefore more "accurate" the computational model is.

## 7.3 Addressing Discrepancies

If discrepancies exist, the steps taking to determine and correct them are as follows:

- Check the governing equations. The equations are similar in both the numerical and analytical method, thus any mistakes may have been carried through to the numerical and analytical solutions, and overlooked in the verification stage. Changes may need to be applied to these equations, and must be re-verified.
- Check the assumptions. All assumptions must be carefully examined. For example, applying boom idealization theory may create too large an error since the shear stress created by the stringer cross section may be more significant than it was initially assumed. Therefore, re-calculating the output but by a numerical method where boom idealization is not applied should be performed, and the discrepancy between it and the experimental result should be found (i.e. the sensitivity of it is found). This sensitivity optimization method should be performed for all assumptions, until the % error discrepancy is reduced as much as possible. Another example of an assumption that is over simplifying is that the total weight is modeled as a uniformly distributed load. In reality, the load will not be uniformly distributed due to the wings and material imperfections. Re-modeling the weight distribution, finding the sensitivity and optimizing for a reduced % error discrepancy, with respect to all other assumptions, should therefore be performed.
- Programming errors. Even after verification, errors in the code might still exist. The program should be thoroughly re-checked.
- Validation errors. Mistakes in calculating the % error, or in comparing the numerical and experimental data might have occurred, and should be checked.
- Experimental data errors. Errors are inevitable in any experiment, caused by inaccuracies or human error. They may exist in the data which will be received. These would make the comparison between the numerical and experimental data not realistic. For

the case of this project, the data will be assumed as correct, since no more information is stated about how it was acquired. For future tests, however, it is important to distinguish errors in the experimental data, classify them as either random (precision) or systematic (bias), and then repeat the experiment as necessary.

Finally, after finding and addressing the reason for the discrepancies, the validity of the model can be determined. This forms the concluding phase of the validation, where the program may be deemed as valid.

## References

- [1] GANTTPROJECT.BIZ. Ganttproject: free desktop project management app, 2016.
- [2] E. Mooij and Z. Papp. *Simulation, Verification and Validation, Lecture Notes*. TU Delft, 2015.
- [3] E. Mooij and Z. Papp. *Structures assignment AE3212 SVV 2016*. TU Delft, 2016.
- [4] R. C. Hibbeler. *Engineering Mechanics: Statics*. Pearson, 2011.
- [5] T. H. G. Megson. *Aircraft Structures for Engineering Students*. Butterworth-Heinemann, Elsevier, 2007.
- [6] Aerostudents.com. Bending, shear and torsion. URL: <http://aerostudents.com/files/aircraftStructuralAnalysis/bendingShearAndTorsion.pdf>.

Group number:

Reviewed by:

Simulation Plan grading sheet		
Items	1 / 0.5 / 0	Page number of comment in pdf
<b>Report overview:</b>	/2	
Task division		
Overall layout		
<b>General:</b>	/4	
Introduce problem		
Identify output variables		
General Assumptions		
Reference Frames		
<b>Analytical part:</b>	/2	
Assumptions		
Relevant equations		
<b>Numerical part:</b>	/3	
Assumptions		
Relevant equations		
Numerical methods		
<b>Flow Chart:</b>	/1	
Flowchart of proposed code (with I/O)		
<b>Verification:</b>	/3	
Proposed unit tests		
Proposed system test		
Plan for addressing discrepancies (errors)		
<b>Validation:</b>	/2	
Proposed validation tests		
Plan for addressing discrepancies (related to assumptions)		

The above points are no grades, they are used for internal purposes.  
The pass/fail is based on the presence of all major elements