Waterloo Collegiate Institute

SPH-3UW

AP Physics 1

Oscillation Lab

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January 7, 2016

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Abstract

The purpose of this lab was to illustrate the validity of the law of conservation of energy along with the determination of the spring constant of a given spring. For the first part the spring constant k was to be found from a given spring. Through the suspension of various known metal masses on a vertically suspended spring, the spring constant was determined. Two methods were used: the algebraic rearrangement of Hooke's Law and a slope analysis of a linear regression on a Force (N) against Stretch Length (m) scatter plot. The spring constant k was determined to be 26.438 ± 1.063. For the second part of the lab, the aim was to validate the law of conservation of energy through the oscillation of a vertically suspended spring. Data was collected using a Vernier Motion Detector 2 machine and the various energies (kinetic energy, gravitational potential energy and spring potential energy) were collected and summed up. The sum of these energies yielded a fairly constant energy total (2.287 J $\pm$ 0.025 J) which supports the authenticity of the law of conservation of energy. While there were some uncertainties due to the lab setup, human error and equipment error it did not affect the validity of the methods during experimentation. Overall, the spring constant k of a given spring was determined and the law of conservation of energy was validated through the calculation of total energy during a suspended mass' oscillation.

1 Introduction

Despite springs not being a considered one of the simple machines, they still have a plethora of uses. Some uses of springs include: mattress bases, sofa seats, hydraulic gears, non-digital clocks and wrist-watches. A spring is typically a helical metal coil which can be compressed or pulled, but always attempt to return to their initial position when released. Springs are one of the few mechanisms in the world which are conservative forces; that is to say that other forms of energy can be converted, stored and released again without the dissipation of any energy. Hooke's Law states that the force needed to extend or compress a spring by some distance is proportional to that distance and it allows for the calculation of any spring constant with displacement, mass and gravity measurements. Furthermore, Antoine Lavoisier, a French chemist, discovered the Law of Conservation of Mass which eventually paved the path for Julius Robert Mayer to discover the Law of Conservation of Energy. This lab aims to determine the spring constant of a given spring (procedure can be applied to virtually any spring). Additionally, this lab confirms the validity of the Law of Conservation of Energy through the calculation of total energy (the sum of kinetic energy, gravitational potential energy and spring potential energy). All in all, springs are a unique mechanism which can be utilized for a variety of purposes, one of which includes a method to confirm the authenticity of the Law of Conservation of Energy.

2 Equipment List

- 1 balance beam (scale)
- 1 pen
- 1 metal spring
- 7 metal weights
- 1 meter stick
- Motion Detector 2 (Vernier)
- 1 retort stand
- 1 roll of masking tape
- Texas Instruments Ti-Nspire Cx Handheld Graphing Calculator

Part I

Part 1: Finding the Spring Constant k

3 Background

The spring constant k was discovered by British physicist, Robert Hooke in 1676. Hooke went on to create Hooke's Law which stated that the displacement in stretch length of the spring was proportional to some spring constant k. There are two primary methods in determining the spring constant, algebraically determining it using Hooke's Law or using a linear regression to model stretch length against the force of gravity. Both methods give a fairly accurate estimate of the spring constant of the given spring.

4 Procedure

1. Gather all necessary equipment and ensure that there is a safe working environment.
2. Measure the mass of each given weight using the balance beam and record masses.
3. Attach the meter stick to the lengthwise portion of the retort stand. Ensure to keep this perpendicular to the ground surface.
4. Attach spring to the retort stand, ensuring to keep the spring parallel to the meter stick.
5. Take photographs of setup, in case of necessity for future reference.
6. Record measurements of initial spring position (initial equilibrium position and initial stretch length).
7. Attach the first metal weight, while recording the corresponding new stretch length and the displacement from the spring equilibrium position.
8. Repeat step 7 with various combinations of given masses, until all combinations have been used.
9. Collect equipment and clean up work area.

5 Experimental Setup

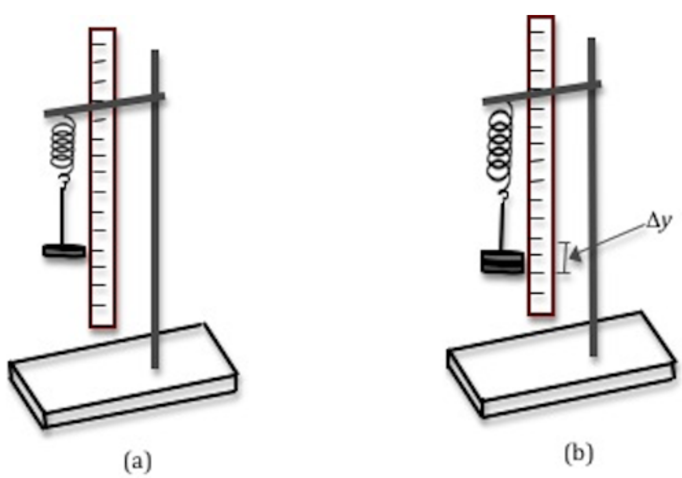


Figure 1: Theoretical Experiment Apparatus Setup for Part 1 (Determining the Spring Constant)

All measurements were taken with respect to the base of the retort stand. The measurements taken from the apparatus included the distance to the top of the spring, the initial bottom of the spring and the new bottom of spring (after the mass was put on the spring). Note that Δy was not actually used as a variable, it is merely to show the displacement of the spring after a mass has been suspended. Shown in Figure 2 is the practical apparatus setup for use in the experiment.

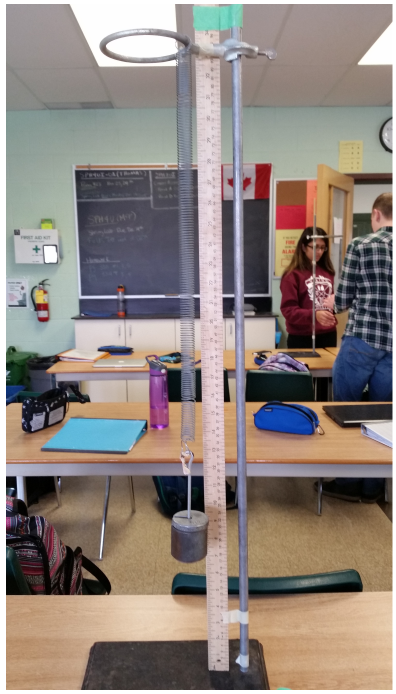


Figure 2: Experiment Apparatus Setup for Part 1 (Determining the Spring Constant)

6 Data and Analysis

The spring constant k was able to determined after taking the previously outlined measurements. After collecting the data required, there were two methods which could be employed to determine the spring constant. Applying the measurements individually to Hooke's Law or utilizing linear regression are both methods which can determine the spring constant k.

6.1 Linear Regression Method

The linear regression method utilizes the knowledge that graphing force against stretch length yields a slope corresponding to the spring constant, k. This occurs due to the fact that the slope is calculated by dividing Δy by Δx ($m = \frac{\Delta y}{\Delta x}$), which is essentially the change in force divided by the change in stretch length (essentially being the displacement). Thus, it is through the use of Hooke's Law that the spring constant could be determined. However, the result will not be the exact, albeit similar, to the result determined algebraically (rearranging Hooke's Law) as the trendline is a line of best fit, an approximation.

6.1.1 Experimental Results

To determine the slope of a graph of Force Exerted by Spring (N) against Stretch Length (m), first the Force Exerted by the Spring must be determined. Recalling on Newton's second law (in this case acceleration is 0), $\Sigma F = 0$. Since the only two forces acting on the object are the force of gravity and the force exerted by the spring, the force exerted by the spring must be equal to the weight of the object.

$$\begin{aligned} \Sigma F &= 0 \\ F_S - F_G &= 0 \\ F_S &= F_G \\ F_S &= mg \end{aligned}$$

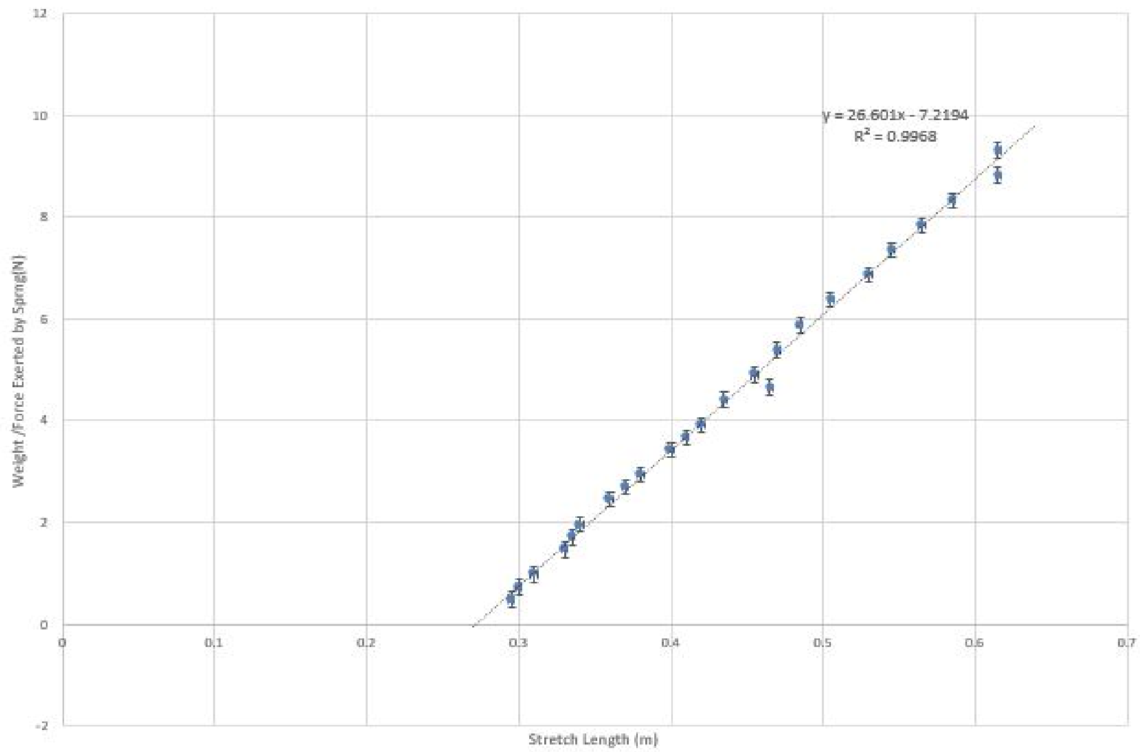
Additionally, the stretch length can be calculated by determining the distance from the top of the spring to the bottom of the spring. Table 1 summarizes these measurements and the respective calculations.

Table 1: Stretch Length of Spring for Various Metal Weights

Trial	**Weight(N)**	**Top of Spring Position(m)**	**Position Final(m)**	**Stretch Length(m)**
1	0.490	0.840	0.545	0.295
2	0.735	0.840	0.540	0.300
3	0.980	0.840	0.530	0.310
4	1.470	0.840	0.510	0.330
5	1.715	0.840	0.505	0.335
6	1.960	0.840	0.500	0.340
7	2.450	0.840	0.480	0.360
8	2.695	0.840	0.470	0.370
9	2.940	0.840	0.460	0.380
10	3.430	0.840	0.440	0.400
11	3.675	0.840	0.430	0.410
12	3.920	0.840	0.420	0.420
13	4.410	0.840	0.405	0.435
14	4.655	0.840	0.375	0.465
15	4.900	0.840	0.385	0.455
16	5.390	0.840	0.370	0.470
17	5.880	0.840	0.355	0.485
18	6.370	0.840	0.335	0.505
19	6.860	0.840	0.310	0.530
20	7.350	0.840	0.295	0.545
21	7.840	0.840	0.275	0.565
22	8.330	0.840	0.255	0.585
23	8.820	0.840	0.225	0.615
24	9.310	0.840	0.225	0.615

For a more thorough presentation of the data collected, please refer to Appendix A. Figure 3 shows the graph (created using Microsoft Excel 2013) of Weight (N) against Stretch Length (m), the equation of the trendline and the correlation value (R^2 value) between the data points and the trendline. Note that the graph The graph results in a line of format $y = mx + b$ as opposed to the ideal y=mx format (since $F_S = kx$). The significance of this will be further discussed in the Theoretical Analysis section. As seen on the graph the equation of the line is $y = 26.601x - 7.2194$.

Figure 3: Weight (N) of Various Suspended Masses on a Spring



6.1.2 Sample Calculations

Below are all the calculations for the Trial 7 (as shown in Figure 3 and Table 1).

$$F_G = mg$$
$$F_G = (0.250kg)(9.8\frac{m}{s^2})$$
$$F_G = 2.450N$$

It is important to note that $F_G = F_S$, that is to say that the weight of the object is equal to the force exerted by the spring. The variable S represents the stretch Length, the variable P_T represents the position of the top of the spring with respect to the bottom of the retort stand and the variable P_F represents the final position after the mass was placed on the spring.

$$S = P_T - P_F$$
$$S = 0.840m - 0.480m$$
$$S = 0.360m$$

6.1.3 Theoretical Analysis

Utilizing a linear regression to determine the value of the spring constant k allows for one vital piece of information to be extrapolated from the created graph. Before determining this key piece of information, a fundamental property of real springs must be discussed. There are three types of

springs; ideal springs, compression springs and extension springs. An ideal spring cannot be created as it is a "perfect spring," exerts a completely constant force as stretched or compressed and is completely massless. A compression spring is one which offers resistance when being compressed. An extension spring on the other hand, is one which offers resistance when being stretched. An extension spring was tested in this lab and thus it is evident that the true initial equilibrium stretch length is larger than the relaxed stretch length of the spring.
The equation of the line of best fit in the graph of Weight against Stretch Length (Figure 3), is in the form $y = mx + b$ and it is known that the force on the spring by gravity (weight) is equal to the force exerted by the spring. Thus, it is clear that the x-intercept (when the force by gravity is zero-and consequently so is the force exerted by the spring) indicates the stretch length of the spring when it is at its true equilibrium position.
Since the variable b is a constant it does not affect the variable m. Removing b results in the equation, $F_S = ms$ ($F_S = F_G = mg$) which correlates with Hooke's Law of $F_S = kx$. It is important to note that the slope of the regression line is the spring constant regardless of whether the displacement from the spring's equilibrium position or the spring's actual length. Both these values change at the same rate with respect to the force exerted by the spring, which again indicates that a translation (albeit horizontal this time) does not impact the calculation of the spring constant.
The slope of the regression line was calculated using Microsoft Excel's trendline function, and below is the equation used to calculate the slope of a linear regression:

$$m_{regression} = \frac{\sum_{i=1}^{n}[(x_i - \bar{x})(y_i - \bar{y})]}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{1}$$

The x-values are represented by the Stretch Length and the y-values are represented by the Weight. There were a total of 24 trials and all were included in the calculation of this regression line.

$$m_{regression} = \frac{\sum_{n=1}^{24}[(x_i - \bar{x})(y_i - \bar{y})]}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{2}$$

The slope was calculated to be 26.601, and thus the spring constant k can be determined to be 26.601 $\frac{N}{m}$. Refer to Appendix A for the calculation of the regression line.

6.2 Hooke's Law Method

Measurements of spring displacement, mass and gravity can be taken and applied to Hooke's Law (see Equation 4) in order to determine the missing variable, the spring constant k. Further information regarding Hooke's Law and its use will be outlined in the Theoretical Analysis section. The recorded measurements, and calculations are all outlined below.

6.2.1 Experimental Results

It is important to recall that the spring used for this lab was an extension spring; one with internal tension. The displacement cannot be calculated with respect to the relaxed position of the spring, but the equilibrium position of the spring. Recalling from the previous method, $y = 26.601x - 7.2194$. Since the y-axis actually represents the force of gravity on the mass (weight) and thus the force exerted by the spring ($F_S = F_G$), when set to 0 the stretch length of the spring at its true equilibrium can be determined.

The initial equilibrium position of the spring occurs when the stretch length of the spring is 0.271m. Furthermore, in order to determine the distance from the base of the retort stand, the stretch length of the spring must be subtracted from the height of the top of the spring (0.84m), which would give the height from the base of the retort stand to the bottom of the spring at its true equilibrium position (0.569m).

$$\begin{aligned} y &= 26.601x - 7.2194 \\ 0 &= 26.601x - 7.2194 \\ x &= \frac{7.2194}{26.601} \\ x &= 0.271 \end{aligned}$$

$$\begin{aligned} h &= P_T - x \\ h &= 0.840m - 0.271m \\ h &= 0.569m \end{aligned}$$

Hooke's Law can be rearranged to solve for the spring constant k and then combined with the equation for displacement.

$$\begin{aligned} F_S &= F_G \\ kx &= mg \\ k &= \frac{mg}{x} \end{aligned}$$

$$x = h_{eq} - h$$

$$k = \frac{mg}{h_{eq} - h} \tag{3}$$

Table 2 summarizes the measurements taken for this part of the lab (with the correct new equilibrium position). The spring constant was also calculated using Hooke's Law for each data point (without taking into account uncertainty). The spring constant k was calculated to be $26.274\frac{N}{m}$.

Table 2: Spring Displacement from Equilibrium with Various Suspended Masses

Trial Number:	**Force Exerted by Spring(N):**	**Displacement(m):**	**Spring Constant(N/m):**
1	0.490	0.024	20.41666667
2	0.735	0.029	25.34482759
3	0.980	0.039	25.12820513
4	1.470	0.059	24.91525424
5	1.715	0.064	26.796875
6	1.960	0.069	28.4057971
7	2.450	0.089	27.52808989
8	2.695	0.099	27.22222222
9	2.940	0.109	26.97247706
10	3.430	0.129	26.58914729
11	3.675	0.139	26.43884892
12	3.920	0.149	26.30872483
13	4.410	0.164	26.8902439
14	4.655	0.194	23.99484536
15	4.900	0.184	26.63043478
16	5.390	0.199	27.08542714
17	5.880	0.214	27.47663551
18	6.370	0.234	27.22222222
19	6.860	0.259	26.48648649
20	7.350	0.274	26.82481752
21	7.840	0.294	26.66666667
22	8.330	0.314	26.52866242
23	8.820	0.344	25.63953488
24	9.310	0.344	27.06395349
Average:	**N/A**	**N/A**	**26.27404443**

6.2.2 Sample Calculations

The following is a sample calculation using the measurements taken for Trial 7. The uncertainty in each measurement is not accounted for in this calculation, but will be taken into account later on.

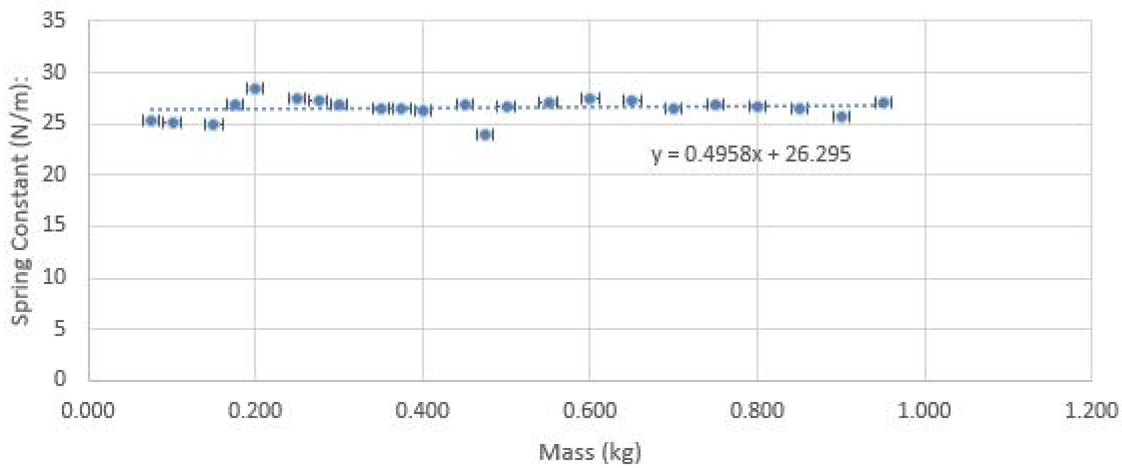
$$\begin{aligned} k &= \frac{mg}{h_{eq} - h} \\ k &= \frac{0.250kg * 9.8\frac{m}{s^2}}{0.569m - 0.480m} \\ k &= 27.528\frac{N}{m} \end{aligned}$$

Please refer to Appendix A for further clarification regarding calculations and data for all trials in Excel 2013.

6.2.3 Theoretical Analysis

In the previous section, Hooke's Law was rearranged and was combined with the displacement equation which resulted in, $k = \frac{mg}{h_{eq} - h}$. The graph below is the determined spring constant k with respect to each suspended mass, and as expected the slope is very close to 0 (a horizontal line) as the value is constant. Trial 1 was excluded as it was deemed to be an outlier. Also, please not that the vertical error bars are present, however are two small to be visually recognized.

Figure 4: Spring Constants with Respect to Various Masses



A common mistake is to assume that the relaxed length of a spring is equal to the equilibrium length of the spring. Recalling the impracticality of an ideal spring (massless and exerts a perfectly constant force), the fact that the spring used was an extension spring then it becomes clear that the true equilibrium length would be longer than the relaxed length of the spring. This was accounted for utilizing the knowledge gained from the previous section using the linear regression and finding the x-intercept (where there is no force exerted by the spring). Should the relaxed length of the spring be used, the resultant graph (incorrect spring constant values against mass) would be an exponential function with a positive horizontal asymptote. This asymptote ($\lim_{x\to\infty}$) represents the correct spring constant k. While the correct spring constant can still be found in the graph of the erroneous spring constant values, it is much more complex and unnecessary since it is much simpler to use the correct equilibrium position.

7 Discussion

7.1 Error Analysis and Uncertainties

7.1.1 Error Analysis-Linear Regression Method

First, the uncertainty using the linear regression method will be calculated using standard deviation and a two tailed t-test. To calculate the standard deviation and therefore the error for the spring constant, the standard deviations of the y values must first be obtained through the following equation:

$$\sigma_{y/x} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2}} \tag{4}$$

Using Microsoft Excel, the standard deviation for the values of the force exerted by the spring is 0.148306382. The standard deviation of the slope is then calculated as:

$$\sigma_m = \frac{\sigma_{y/x}}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}} \tag{5}$$

Essentially, the standard deviation of the slope is equal to the standard deviation of the y-values divided by the standard deviation of the x-values. Using Microsoft Excel's built in statistics calculator, the standard deviation for the slope of the regression line was calculated to be 0.635326497. The confidence limit for the slope of a regression line is written as $m \pm (t_n \sigma_m)$ where t_n represents the two tailed t-statistical value for n degrees of freedom. The uncertainty amount (margin of error) is then calculated with 23 degrees of freedom and a 95 percent confidence level:

$$\begin{aligned} \Delta k &= t_n \sigma_m \\ \Delta k &= (2.06865761)(0.635326497) \\ \Delta k &= 1.31427299285 \end{aligned}$$

The degrees of freedom were calculated based on the fact that the data is paired, any times it makes sense to treat data as paired. It is typically assumed that if the sample has a total of n pairs of data points then there are n - 1 degrees of freedom (for paired data). After multiplying the expected t-value with the standard deviation of the slope of the line, the spring constant obtained from this method is $(26.601 \pm 1.314 \frac{N}{m})$. Furthermore the slope of the regression line has a correlation value of 99.8%, which validates the accuracy of the measurements and validates the method to calculate the spring constant k.

7.1.2 Error Analysis-Hooke's Law Method

Secondly, partial derivatives were used to propagate the uncertainty when using Hooke's Law and algebraically determining the spring constant.The below equation is the undifferentiated form of the equation to determine the final uncertainty.

$$\Delta k = \sqrt{\left(\frac{\partial}{\partial m}\Delta m\right)^2 + \left(\frac{\partial}{\partial h}\Delta h\right)^2 + \left(\frac{\partial}{\partial h_{eq}}\Delta h_{eq}\right)^2} \tag{6}$$

$$\Delta k = \sqrt{\left(\frac{g}{(h_{eq} - h)}\Delta m\right)^2 + \left(-\frac{mg}{(h_{eq} - h)^2}\Delta h\right)^2 + \left(\frac{mg}{(h_{eq} - h)^2}\Delta h_{eq}\right)^2} \tag{7}$$

The following table summarizes the each trial and their respective uncertainties. It is important to note that the following uncertainties were assumed based on judgment and machine accuracy. Please refer to Appendix B for a more complete presentation on uncertainty calculations.

$$\begin{aligned} \Delta m &= \pm 0.003kg \\ \Delta h &= \pm 0.002m \\ \Delta h_{eq} &= \pm 0.002m \end{aligned}$$

Table 3: Trials and Their Respective Final Uncertainties

Trial #	**Final Uncertainty**
1	2.700013574
2	2.671744584
3	1.972155538
4	1.294200614
5	1.270240434
6	1.239911989
7	0.935134314
8	0.832506787
9	0.750079319
10	0.625952649
11	0.578072746
12	0.536977849
13	0.497205222
14	0.381247541
15	0.439438403
16	0.412345866
17	0.388274849
18	0.35221441
19	0.310724028
20	0.296967434
21	0.275347413
22	0.256651533
23	0.227478053
24	0.238372459
Average:	**0.8118024**

Including all trials resulted in a final uncertainty value of ± 0.811. Thus, the spring constant can be deemed to be $26.274\ \frac{N}{m} \pm 0.811\frac{N}{m}$.

7.2 Closure of Part 1

Averaging the values and uncertainties from both methods (Linear Regression and Hooke's Law), the final result can be determined.

$$\begin{aligned} k &= \frac{26.601 + 26.274}{2} \pm \frac{1.314 + 0.811}{2} \\ k &= 26.438 \pm 1.063 \end{aligned}$$

The relative error is:

$$\begin{aligned} RelativeError &= \frac{\Delta k * 100\%}{k} \\ RelativeError &= \frac{1.063\frac{N}{m} * 100\%}{26.438\frac{N}{m}} \\ RelativeError &= 4.02072774\% \end{aligned}$$

The relative error is 4.021%.

7.2.1 Future Steps and Applications

All in all, the spring constant was determined to be $k = 26.438 \pm 1.063$. The uncertainties taken were based on judgment and were on the generous side. While the uncertainty is not minuscule, it demonstrates that conventional methods using simple equipment allows for an accurate way to determine the spring constant k. This can be applied to a variety of industrial locations as springs are a very common mechanism. Of course, the use of more precise equipment would greatly improve the accuracy of the result. The uncertainties of length/height measurements and mass measurements could be decreased with more precise measuring equipment such as a better scale or computer based distance measurements. Additionally, the spring used had a curled loop on either end which meant that determining the actual length of the spring (active component of the spring) is rather difficult. However, the determined value for the spring constant k, 26.438 ± 1.063, is an accurate figure of the spring's true spring constant.

Part II

Part 2: Confirming the Law of Conservation of Energy

8 Background

The law of the conservation of energy states that the total energy of an isolated system must remain constant. In short, energy can not be created or destroyed, only transformed. This section of the lab will analyze the total energy of an oscillating spring system and use the results to demonstrate the law of conservation of energy. 2 methods will be used to prove the law. The first will utilize equation manipulations and rearrangements and the second will utilize energy summations.

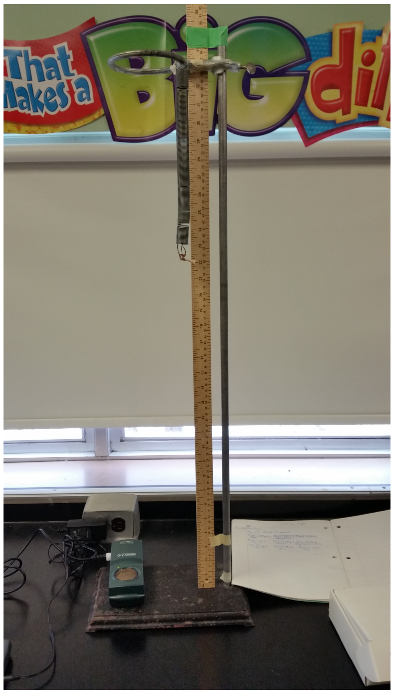
9 Procedure

1. Gather all necessary equipment and ensure that there is a safe working environment.
2. Set up apparatus in the same fashion as in Part 1: Determining the spring constant k.
3. Suspend a mass on the spring and record the mass.
4. Place the Vernier Motion Detector directly below the suspended mass.
5. Determine the height of the spring (both at top and bottom) at its new equilibrium (with suspended mass).
6. Determine the height of the probe.
7. Pull down the mass and let go, allowing for the system to oscillate. Ensure to not put work (add energy) to the system by throwing the mass, instead simply release it after pulling it down.

8. Allow the Vernier Motion Detector to collect data.
9. Collect equipment and clean up work area.

10 Experimental Setup

Figure 5: Experiment Apparatus Setup for Part 2



This image illustrates the experiment apparatus used for the second part of this lab.

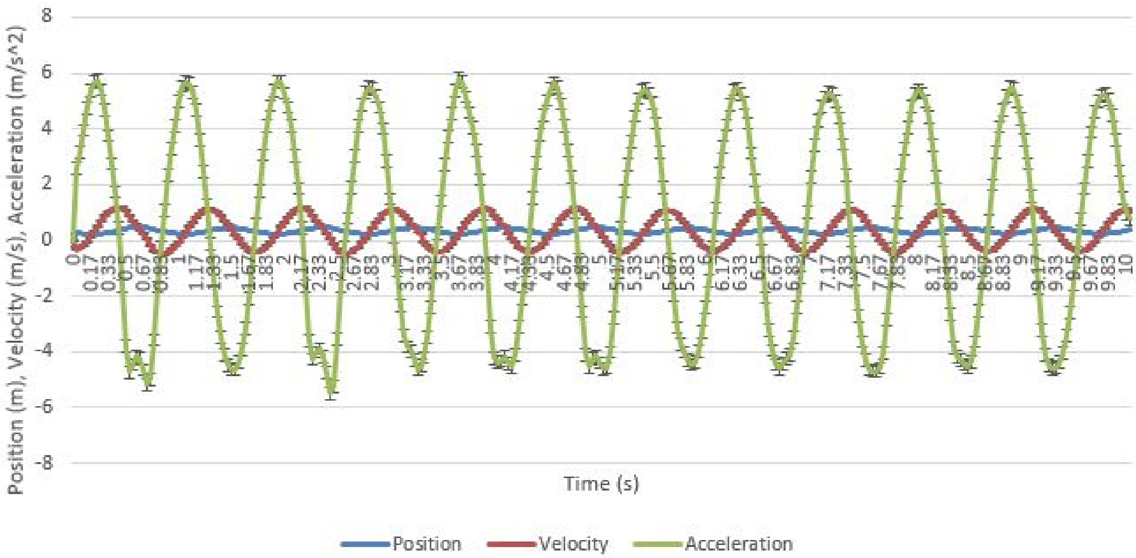
11 Data and Analysis

The data can be interpreted in two primary fashions, one using energy summation and the other algebraically, similarly to the previous part.

11.1 Experimental Results

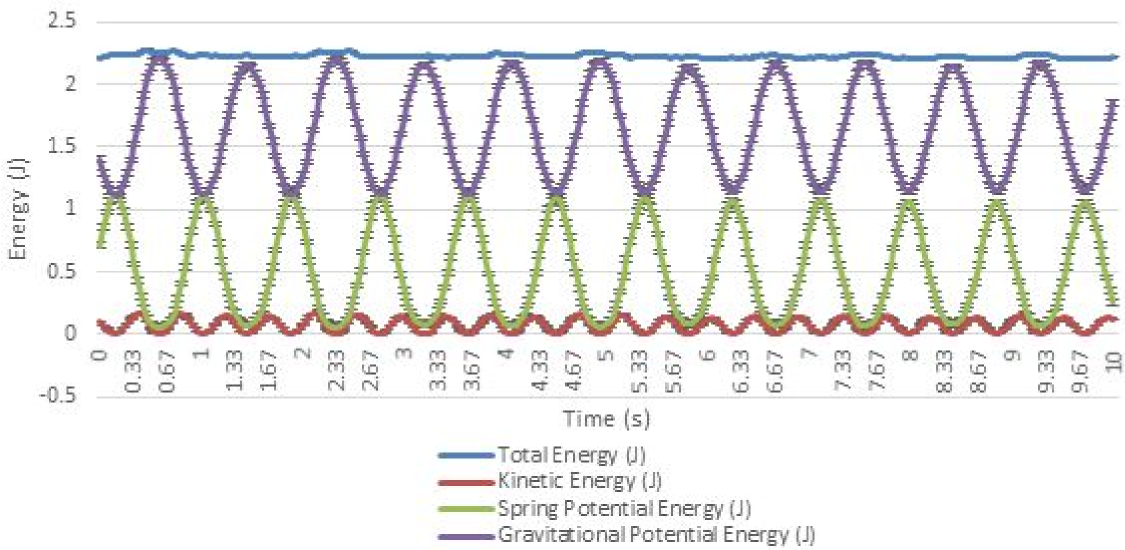
In the graph below, the position, velocity and acceleration at different times during the oscillation is graphed. The graph demonstrates the periodic nature of the position, velocity and acceleration during a spring's oscillation. It is important to note that the velocity is at its maximum when the position is furthest away from the spring's equilibrium and the acceleration is at its maximum when the position is at the spring's equilibrium. All data points were included, but certain acceleration values were deemed outliers attributed machinery error.

Figure 6: Position, Velocity and Acceleration Data from Motion Detector



Furthermore, gravitational potential energy, spring potential energy and kinetic energy can all be calculated with the data collected. The method as to how this was discovered will be discussed more extensively in the Theoretical Analysis section. Visually, it can be observed that the total energy stays consistent, thus enforcing the validity of the law of conservation of energy. Keep in mind that the below graph, does not factor in overall effective mass to the oscillation and instantaneous velocities, but only the mass of the metal weight and velocity given by the Vernier Motion Detector 2. It is important to note that the maximum gravitational energy times correspond with a slight increase in the total energy. This can be attributed to the fact that the retort stand itself might be pushed slightly upward (adding energy to the system) when the mass is at its highest position. Furthermore, the minimum of gravitational potential energy correlates with the minimum of the total energy, likely because the retort stand gets pulled downward when the mass is at its lowest position (lessening total energy). Thus, the total energy vaguely resembles a sinusoidal function. Please refer to Appendix C for a complete overview of the oscillation data collected and Appendix D for the energy components data.

Figure 7: Energy in its Various Components During a Spring's Oscillation (Not including caculated instantaneous velocity values or total effective mass)



11.2 Theoretical Analysis

11.2.1 Algebraic Method

To determine the total energy, each individual form of energy must be accounted for. Below are the defined constants (Equations 9-10), which are then subbed into Equation 11.

$$x_{eq} = l_e q - (h_t - h_b) \quad (8)$$
$$x = x_{eq} + y \quad (9)$$
$$h = h_{eq} + y \quad (10)$$

$$E_T = \frac{1}{2}mv_i^2 + \frac{1}{2}kx^2 + mgh \quad (11)$$

Equation 11 can be expanded using the constants to determine the final expanded and simplified equation.

$$\begin{aligned}
E_T &= \frac{1}{2}mv_i^2 + \frac{1}{2}kx^2 + mgh \\
E_T &= \frac{1}{2}mv_i^2 + \frac{1}{2}k(x_{eq} + y)^2 + mg(h_{eq} + y) \\
E_T &= \frac{1}{2}mv_i^2 + \frac{1}{2}k(x_{eq}^2 + 2x_{eq}y + y) + mgh_{eq} + mgy \\
E_T &= \frac{1}{2}mv_i^2 + \frac{1}{2}kx_{eq}^2 + kx_{eq}y + \frac{1}{2}ky^2 + mgh_{eq} + mgy \\
E_T &= \frac{1}{2}mv_i^2 + \frac{1}{2}k(\frac{-mg}{k})^2 + k(\frac{-mg}{k})y + \frac{1}{2}ky^2 + mgh_{eq} + mgy
\end{aligned}$$

$$E_T = \frac{1}{2}mv_i^2 + \frac{1}{2}ky^2 + mgh_{eq} + \frac{(mg)^2}{2k} \quad (12)$$

Equation 12 is equal to Equation 11, but is more useful as there only two variable terms (variables v and y). This equation is even more useful when $y = h - h_e q$ is subbed in (see Equation 13).

$$E_T = \frac{1}{2}mv_i^2 + \frac{1}{2}k(h - h_{eq})^2 + mgh_{eq} + \frac{(mg)^2}{2k} \tag{13}$$

This equation can be made even more accurate by factoring in the effective mass of the spring itself and the utilization of instantaneous velocity values. Their is some component of the spring during all stages of the oscillation that is contributing to the amplitude of the oscillation. Factoring in the effective mass of the spring and the instantaneous velocity values increases the precision of Equation 13. The instantaneous velocity values can be found by taking the derivative of the sinusoidal regression of position with respect to time ($v(t) = \frac{dx}{dt}$). Using a Texas Instruments Ti-Nspire Cx Handheld Graphing Calculator, the equation of the sinusoidal regression was determined to be:

$$0.10899696981647sin(7.2208080952085x2.6586819947679) + 0.34363304480055 \tag{14}$$

The derivative of this equation is shown below and it gives the instantaneous velocity values at any given point in time.

$$0.7870462020040cos(7.2208080952085x - 2.6586819947679) \tag{15}$$

Equation 13 can be further ameliorated by taking into account the effective mass of the spring itself. The kinetic energy of a vertically oscillating spring of length l is said to be:

$$K = \int_0^l \frac{1}{2}v^2 dm \tag{16}$$

This integral can be solved by assuming homogeneous stretching and uniform mass distribution. Also the velocity is a function of the position x measured from a fixed point on the spring.

$$dm = \frac{m_S}{l}dx \tag{17}$$

$$v(x) = \frac{v_i}{l}x \tag{18}$$

When Equations 17 and 18 are subbed into Equation 16, it yields the following.

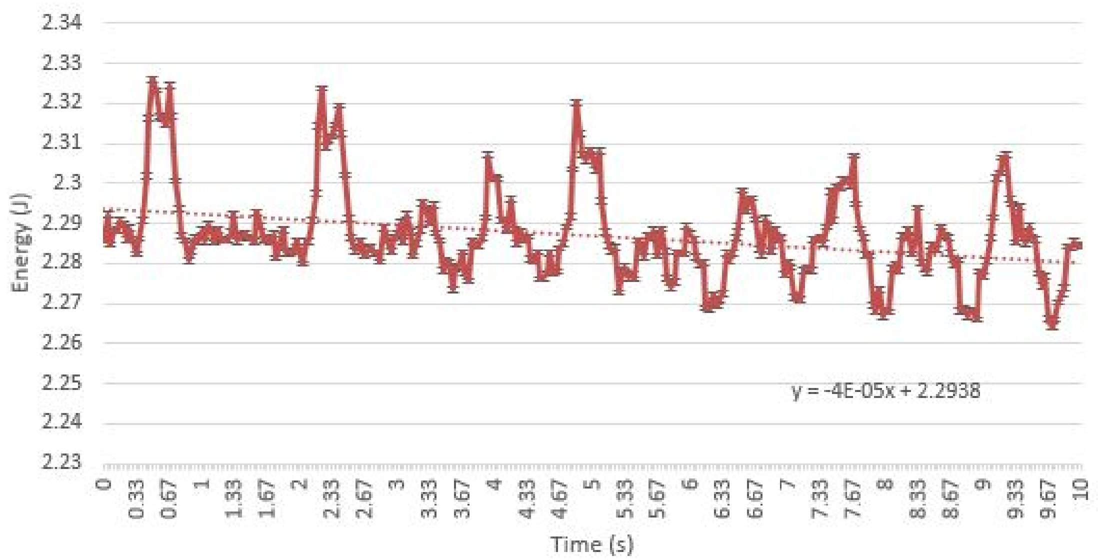
$$K = \frac{1}{2}\frac{m_S}{3}v_i^2 \tag{19}$$

It is now evident that $\frac{1}{3}$ of the mass of the spring contributes to the oscillation of the spring.

$$m = m_{mass} + \frac{m_{spring}}{3} \tag{20}$$

Taking into account the overall effective mass and the instantaneous velocity values, Equation 13 ($E_T = \frac{1}{2}mv_i^2 + \frac{1}{2}k(h - h_{eq})^2 + mgh_{eq} + \frac{(mg)^2}{2k}$) can now be effectively utilized. Please note that the variable m refers to the effective mass to the oscillation and not only the mass of the metal weight. Using this equation, all data the data points were plugged in and the total energies were graphed as seen below. For further information please refer to Appendix D.

Figure 8: Total Energy During a Spring's Oscillation



As seen in the graph, it is clear that the law of conservation is valid since the total energy remains constant. There are deviations, but they will be explained to be insignificant in the Discussion section. Furthermore, there is a downward slope which indicates that some form of dampening was most likely present. This is to be expected as air resistance and other such factors would take energy away from the system, but only plays a minuscule factor. Moreover, it can be recognized that there the total energy resembles sinusoidal oscillation vaguely. This can be attributed to the fact that the spring was exerting a force on the top of the retort stand, pulling it down slightly. This would indicate that as the machine would read the position regardless of the position of the stand, energy would be added to the system.

11.2.2 Energy Summation

The total energy can be shown to be constant by determining the total energy at each point in all of its different forms (gravitational potential, spring potential and kinetic). This can be done by using the below equations for kinetic energy, gravitational potential and spring potential respectively.

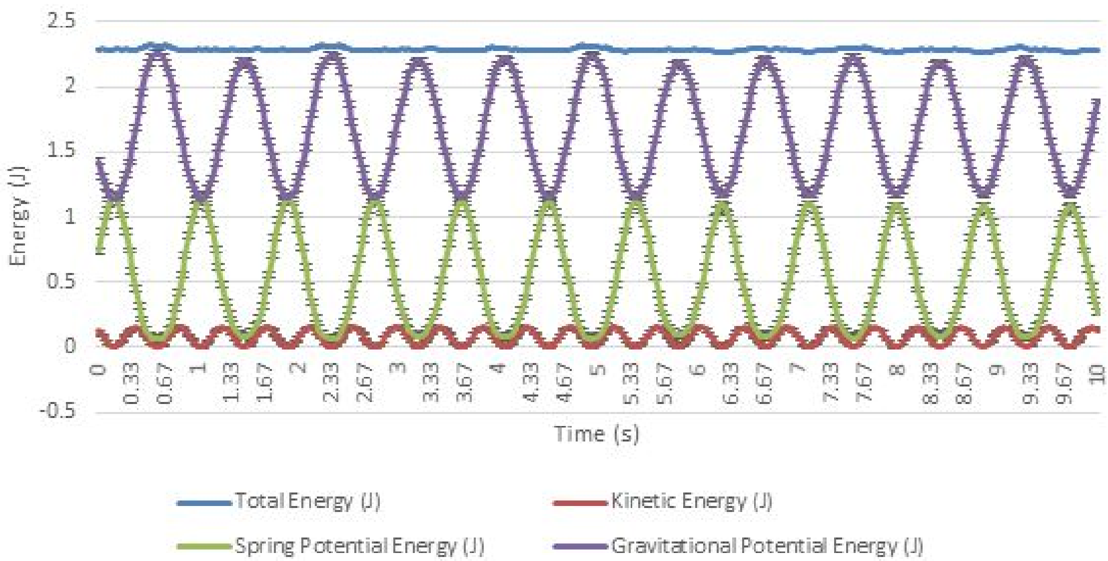
$$KE = \frac{1}{2}mv_i^2 \tag{21}$$

$$U_G = mgh \tag{22}$$

$$U_S = \frac{1}{2}k(\frac{-mg}{k} + h - h_{eq})^2 \tag{23}$$

Refer to the figure below for a visual representation of each component of the total energy. Once again it is evident that energy is conserved in all stages of oscillation. The total energy hovers around 2.287 J, although there is some slight variance (further extended on in the Discussion section). Figure 9 incorporates both the instantaneous velocity values and effective mass.

Figure 9: Energy in is Various Components-Ameliorated Method(Instantaneous Velocities and Effective Mass)



11.3 Sample Calculations

Sample calculations for both methods are using the data when t=0s (a randomly selected point).

11.3.1 Sample Calculations-Algebraic Equation Manipulation

$$E_T = \frac{1}{2}mv_i^2 + \frac{1}{2}k(h-h_{eq})^2 + mgh_{eq} + \frac{(mg)^2}{2k}$$

$$E_T = \frac{1}{2}(0.4924kg + \frac{0.077kg}{3})(-0.69705\frac{m}{s})^2 + \frac{1}{2}(26.438\frac{N}{m})(0.29m - 0.34m)^2 + (0.4924kg + \frac{0.077kg}{3})9.8\frac{m}{s^2}(0.34m) + \frac{(0.4924kg + \frac{0.077kg}{3})(9.8\frac{m}{s^2})^2}{2(26.438\frac{N}{m})}$$

$$E_T = 2.285746438J$$

11.3.2 Sample Calculations-Energy Summation

Kinetic Energy:

$$KE = \frac{1}{2}mv_i^2$$

$$KE = \frac{1}{2}(0.4924kg + \frac{1}{3}(0.077kg))(-0.69705\frac{m}{s})^2$$

$$KE = 0.121970062J$$

Gravitational Potential Energy:

$$U_G = mgh$$

$$U_G = (0.4924kg + \frac{1}{3}(0.077kg))(9.8\frac{m}{s^2})(0.29m)$$

$$U_G = 1.426873467J$$

Spring Potential Energy:

$$\begin{aligned} U_S &= \frac{1}{2}k(\frac{-mg}{k} + h - h_eq)^2 \\ U_S &= \frac{1}{2}(26.438\frac{N}{m})(\frac{-(0.4924kg + \frac{1}{3}(0.077kg))(9.8\frac{m}{s^2})}{26.438\frac{N}{m}} + 0.29m - 0.34m)^2 \\ U_S &= 0.736902909J \end{aligned}$$

Total Energy:

$$\begin{aligned} E_T &= KE + U_G + U_S \\ E_T &= 0.121970062J + 1.426873467J + 0.736902909J \\ E_T &= 2.285746438J \end{aligned}$$

12 Discussion

12.1 Error Analysis and Uncertainties

12.1.1 Error Analysis - Equation Rearrangement

The error for the total energy obtained through equation rearrangement was determined using the partial derivative method.

$$\Delta q = \sqrt{\left(\frac{\partial q}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial q}{\partial x_2}\Delta x_2\right)^2 + \cdots + \left(\frac{\partial q}{\partial x_n}\Delta x_n\right)^2} \tag{24}$$

For use in this scenario:

$$\Delta E_T = \sqrt{\left(\frac{\partial}{\partial m}\Delta m\right)^2 + \left(\frac{\partial}{\partial v}\Delta v\right)^2 + \left(\frac{\partial}{\partial k}\Delta k\right)^2 + \left(\frac{\partial}{\partial h}\Delta h\right)^2 + \left(\frac{\partial}{\partial h_{eq}}\Delta h_{eq}\right)^2}$$

The differentiated equation, taking into account Equation 13:

$$\Delta E_T = \sqrt{\left(\left(\frac{v^2}{2} + \frac{mg^2}{k} + hg\right)\Delta m\right)^2 + (mv\Delta v)^2 + \left(\left(\frac{(h-h_{eq})^2}{2} - \frac{m^2g^2}{2k^2}\right)\Delta k\right)^2 + ((k(h-h_{eq}) + gm)\,\Delta h)^2 + (-k(h-h_{eq})\Delta h_{eq})^2} \tag{25}$$

Uncertainties of 0.02m were taken for length measurements, 0.003kg for mass measurements and 0.003$\frac{m}{s}$ for velocity measurements. When this equation is applied to all data points, a final result of ± 0.022400149 J was determined to be the uncertainty for the equation rearrangement measurement. Please Refer to Appendix E for a complete presentation of all uncertainties at each Data Point (and for a detailed analysis on the calculation of each uncertainty). So the final result using this method is 2.287115426 J ± 0.022400149 J.

12.2 Error Analysis - Energy Summation

The propagation equation was applied to each component of the total energy and then to the formula which summed the three forms of energy.

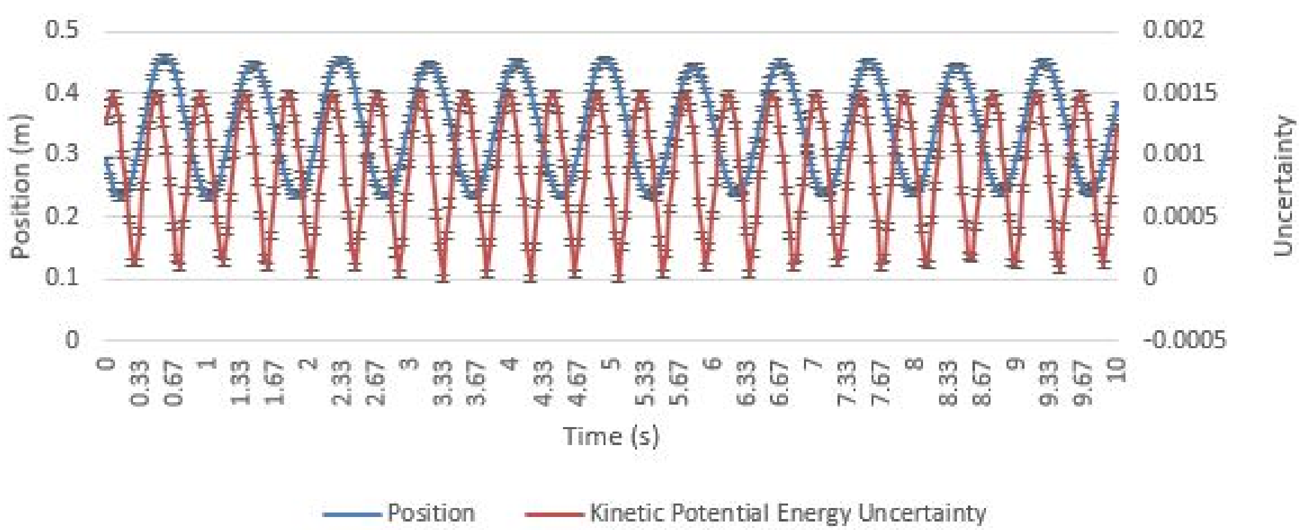
Kinetic Energy:

$$\Delta K = \sqrt{\left(\frac{\partial}{\partial m}\Delta m\right)^2 + \left(\frac{\partial}{\partial v}\Delta v\right)^2}$$

$$\Delta K = \sqrt{\left(\left(\frac{v^2}{2}\right)\Delta m\right)^2 + ((mv)\Delta v)^2}$$

The data was plugged into Microsoft Excel and the resultant graph was produced that summarizes the uncertainty in kinetic energy and its relation to the position of the suspended mass:

Figure 10: Correlation between Uncertainty in Kinetic Energy and Position(Height) of the Mass



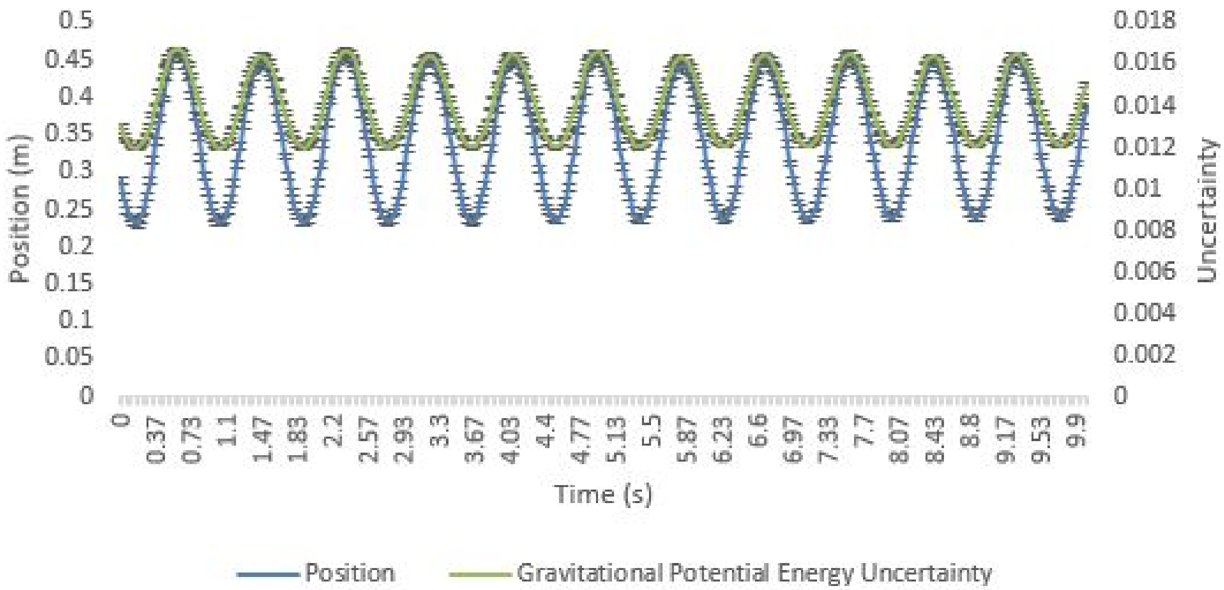
Clearly, as seen in the graph, the maximums and minimums of the sinusoidal regression curve of the position correlate with the minimums of the uncertainties. This makes it evident that there is the least uncertainty when the mass is at its highest or lowest point. It can be concluded that this occurs due to the fact that the velocity is at 0 at these points, and thus the overall uncertainty is minimized. The average uncertainty for kinetic energy is $\pm 0.001008491J$. Gravitational Potential Energy:

$$\Delta U_g = \sqrt{\left(\frac{\partial}{\partial m}\Delta m\right)^2 + \left(\frac{\partial}{\partial h}\Delta h\right)^2}$$

$$\Delta U_g = \sqrt{((gh)\Delta m)^2 + ((mg)\Delta h)^2}$$

Below is the graph correlating the uncertainty with the position of the mass:

Figure 11: Correlation between Uncertainty in Gravitational Potential Energy and Position(Height) of the Mass



The graph demonstrates that the uncertainty is highest when the mass is at its highest point in the oscillation. Furthermore, the uncertainty is at its lowest when the position of the mass is at its lowest. The average uncertainty for gravitational potential energy is $\pm 0.00000129285J$.

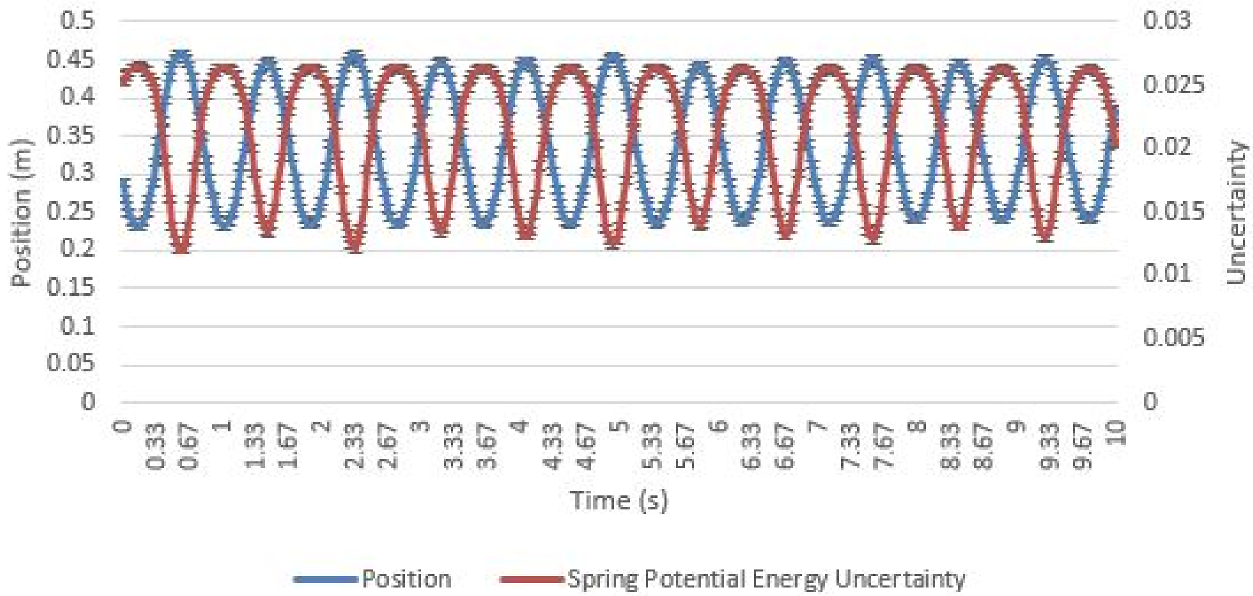
Spring Potential Energy:

$$\Delta U_S = \sqrt{\left(\frac{\partial}{\partial k}\Delta k\right)^2 + \left(\frac{\partial}{\partial m}\Delta m_{eff}\right)^2 + \left(\frac{\partial}{\partial h}\Delta h\right)^2 + \left(\frac{\partial}{\partial h_{eq}}\Delta h_{eq}\right)^2}$$

$$\Delta U_S = \sqrt{\left(\left(\frac{gm\left(-h_{eq} - \frac{gm}{k} + h\right)}{k} + \frac{\left(-h_{eq} - \frac{gm}{k} + h\right)^2}{2}\right)\Delta k\right)^2 + \left(\left(-g\left(-h_{eq} - \frac{gm}{k} + h\right)\right)\Delta m\right)^2 + \left(\left(k\left(-h_{eq} - \frac{gm}{k} + h\right)\right)\Delta h\right)^2 + \left(\left(-k\left(-h_{eq} - \frac{gm}{k} + h\right)\right)\Delta h_{eq}\right)^2}$$

The data was plugged into Microsoft Excel and the resultant graph was produced that summarizes the uncertainty in spring potential energy:

Figure 12: Correlation between Uncertainty in Spring Potential Energy and Position(Height) of the Mass



From this graph, it can be observed that the maximums and minimums of the position function correspond with the minimums of the uncertainty curve. Therefore, the uncertainty in spring potential energy is greatest when the spring is at its highest point. The average uncertainty for spring potential energy is $\pm 0.027945816J$.

Total Energy:

$$\Delta E_T = \sqrt{\left(\frac{\partial}{\partial K}\Delta K\right)^2 + \left(\frac{\partial}{\partial U_g}\Delta U_g\right)^2 + \left(\frac{\partial}{\partial U_S}\Delta U_S\right)^2}$$

$$\Delta E_T = \sqrt{(\Delta K)^2 + (\Delta U_g)^2 + (\Delta U_S)^2}$$

The value for the total energy uncertainty was calculated to be $\pm 0.028192799J$. Thus, the result from this section was determined to be 2.287115426 J $\pm$ 0.028192799 J.

12.3 Closure-Part 2

All in all, the total energy was evidently constant at all points during the spring's oscillation. The final result for total energy was determined to be:

$$E_T = \frac{2.287115426J + 2.287115426}{2} \pm \frac{0.028192799J + 0.022400149J}{2}$$

$$E_T = 2.287115426J \pm 0.025296474J$$

Thus, the total energy was constant throughout the spring's oscillation hovering at about $2.287J \pm 0.025J$. Due to the minuscule standard deviation of the total energy which was calculated to be 0.014699208, it is clear that there is very little variance in total energy. The deviance present can be explained by the factors explained earlier. Those same factors also contributed to the slight dampening effect on the system, where energy was slowly being lost. Furthermore the relative error is displayed below:

$$
\begin{aligned}
RelativeError &= \frac{\Delta E_T * 100\%}{E_T} \\
RelativeError &= \frac{0.025J * 100\%}{2.287115426J} \\
RelativeError &= 1.09307994\%
\end{aligned}
$$

The relative error is 1.093%. Overall the law of conservation of energy was validated using two accurate methods to determine the total energy during a spring's oscillation.

Part III
Closure

13 Conclusion

In conclusion, there were two primary purposes of this lab: to determine the spring constant k of a given spring and to validate the law of the conservation of energy. Two methods were used in order to determine the spring constant: the algebraic rearrangement of Hooke's Law to be used for each point individually or the utilization of a linear regression (on a scatter plot of Force Exerted by Spring/Weight (N) against Stretch Length (m)). Overall, the spring constant k was determined to be 26.438 ± 1.063, factoring in the uncertainty of the method. For the second part of the lab, a mass was suspended by a spring and then put into oscillation. Kinetic energy, gravitational potential energy and spring potential energy were all calculated and summed up to determine the total energy. This calculation was repeated for over a period of ten seconds, where multiple oscillations were completed. Although there were uncertainties, they were primarily from human error, equipment and other measurement problems that would be difficult to greatly improve. In conclusion, this lab was able to determine the spring constant k of a given spring, which could be used to determine the spring potential energy during oscillation of a spring; which was one of the energies which was calculated to validate the law of conservation of energy. All in all, the spring constant k was determined using two methods. Additionally, the law of conservation of energy was validated by measuring the total energy during a spring's oscillation.

14 Acknowledgments

I would like to acknowledge my lab partners: Danish Baig, Kyle Okamoto, Tom Paraschuck and Jack Wallace for their assistance with the conducting of the experimentation for this lab. Furthermore, I would like to express my gratitude to Tiger Jian for his useful input and for introducing me to the wonders of Overleaf. I would also like to thank Mr. Burns for the information and knowledge he provided me which allowed for the completion of this lab. Lastly, I would like to thank the producers of Microsoft Excel, Google and Overleaf for assisting with the production of this lab.

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16 Appendices

16.1 Appendix A: Data Collected for Part 1 and Calculations to Determine Spring Constant k

Figure 13: Data Collected and Calculations to Determine the Spring Constant

Trial	Mass (kg)	Weight (N)	Position of Top Of Spring (m)	Equilibrium Position (m):
1	0.050	0.490	0.840	0.569
2	0.075	0.735	0.840	0.569
3	0.100	0.980	0.840	0.569
4	0.150	1.470	0.840	0.569
5	0.175	1.715	0.840	0.569
6	0.200	1.960	0.840	0.569
7	0.250	2.450	0.840	0.569
8	0.275	2.695	0.840	0.569
9	0.300	2.940	0.840	0.569
10	0.350	3.430	0.840	0.569
11	0.375	3.675	0.840	0.569
12	0.400	3.920	0.840	0.569
13	0.450	4.410	0.840	0.569
14	0.475	4.655	0.840	0.569
15	0.500	4.900	0.840	0.569
16	0.550	5.390	0.840	0.569
17	0.600	5.880	0.840	0.569
18	0.650	6.370	0.840	0.569
19	0.700	6.860	0.840	0.569
20	0.750	7.350	0.840	0.569
21	0.800	7.840	0.840	0.569
22	0.850	8.330	0.840	0.569
23	0.900	8.820	0.840	0.569
24	0.950	9.310	0.840	0.569
Average	N/A	N/A	N/A	N/A

Position Final (m)	Stretch Length (m)	Displacement:	Spring Constant (N/m):
0.545	0.295	0.024	20.41666667
0.540	0.300	0.029	25.34482759
0.530	0.310	0.039	25.12820513
0.510	0.330	0.059	24.91525424
0.505	0.335	0.064	26.796875
0.500	0.340	0.069	28.4057971
0.480	0.360	0.089	27.52808989
0.470	0.370	0.099	27.22222222
0.460	0.380	0.109	26.97247706
0.440	0.400	0.129	26.58914729
0.430	0.410	0.139	26.43884892
0.420	0.420	0.149	26.30872483
0.405	0.435	0.164	26.8902439
0.375	0.465	0.194	23.99484536
0.385	0.455	0.184	26.63043478
0.370	0.470	0.199	27.08542714
0.355	0.485	0.214	27.47663551
0.335	0.505	0.234	27.22222222
0.310	0.530	0.259	26.48648649
0.295	0.545	0.274	26.82481752
0.275	0.565	0.294	26.66666667
0.255	0.585	0.314	26.52866242
0.225	0.615	0.344	25.63953488
0.225	0.615	0.344	27.06395349
N/A	N/A	N/A	26.27404443

Figure 14: Data Collected and Calculations to Determine the Spring Constant-Linear Regression Method

Trial #	Stretched Lengtl	Length - Averag	(Length - Average)^2(	Force Exerted by Spri	Force - Averag	(Force - Average)^2(	(Length - Average)(Force - Av	Regression Resi	(Regression Residual)^2
1	0.2950	-0.143333333	0.020544444	0.490000	-3.950625	15.60743789	0.56625625	-0.14	0.018998916
2	0.3000	-0.138333333	0.019136111	0.735000	-3.705625	13.73165664	0.512611458	-0.03	0.000667747
3	0.3100	-0.128333333	0.016469444	0.980000	-3.460625	11.97592539	0.444113542	-0.05	0.002194857
4	0.3300	-0.108333333	0.011736111	1.470000	-2.970625	8.82461289	0.321817708	-0.09	0.007897218
5	0.3350	-0.103333333	0.010677778	1.715000	-2.725625	7.42903164	0.281647917	0.02	0.000534972
6	0.3400	-0.098333333	0.009669444	1.960000	-2.480625	6.15350039	0.243928125	0.14	0.018258823
7	0.3600	-0.078333333	0.006136111	2.450000	-1.990625	3.96258789	0.155932292	0.09	0.008669141
8	0.3700	-0.068333333	0.004669444	2.695000	-1.745625	3.04720664	0.119284375	0.07	0.00519837
9	0.3800	-0.058333333	0.003402778	2.940000	-1.500625	2.25187539	0.087536458	0.05	0.002610313
10	0.4000	-0.038333333	0.001469444	3.430000	-1.010625	1.02136289	0.038740625	0.01	8.23417E-05
11	0.4100	-0.028333333	0.000802778	3.675000	-0.765625	0.58618164	0.021692708	-0.01	0.000142427
12	0.4200	-0.018333333	0.000336111	3.920000	-0.520625	0.27105039	0.009544792	-0.03	0.001085225
13	0.4350	-0.003333333	1.11111E-05	4.410000	-0.030625	0.00093789	0.000102083	0.06	0.003369164
14	0.4650	0.026666667	0.000711111	4.655000	0.214375	0.04595664	0.005716667	-0.49	0.245006183
15	0.4550	0.016666667	0.000277778	4.900000	0.459375	0.21102539	0.00765625	0.02	0.000256881
16	0.4700	0.031666667	0.001002778	5.390000	0.949375	0.90131289	0.030063542	0.11	0.011452159
17	0.4850	0.046666667	0.002177778	5.880000	1.439375	2.07180039	0.067170833	0.20	0.039204798
18	0.5050	0.066666667	0.004444444	6.370000	1.929375	3.72248789	0.128625	0.16	0.024331327
19	0.5300	0.091666667	0.008402778	6.860000	2.419375	5.85337539	0.221776042	-0.02	0.000362378
20	0.5450	0.106666667	0.011377778	7.350000	2.909375	8.46446289	0.310333333	0.07	0.005176952
21	0.5650	0.126666667	0.016044444	7.840000	3.399375	11.55575039	0.4305875	0.03	0.000896047
22	0.5850	0.146666667	0.021511111	8.330000	3.889375	15.12723789	0.570441667	-0.01	0.000145998
23	0.6150	0.176666667	0.031211111	8.820000	4.379375	19.17892539	0.773689583	-0.32	0.102469412
24	0.6150	0.176666667	0.031211111	9.310000	4.869375	23.71081289	0.86025625	0.17	0.028863143
Average	**0.4383**			**4.4406**					
Sum			**0.233433333**			**165.70651563**	**6.209525**		**0.52787479**
							Slope of Regression Line	26.60084964	
							Intercept of Regression Line	-7.21941409	
							Standard Deviation of Force	0.148306382	
							Standard Deviation of Slope (k	0.635326497	
							R	0.998405929	
							R^2	0.996814399	

16.2 Appendix B: Uncertainty Calculations for Part 1-Hooke's Law Method

Figure 15: Data Collected and Calculations to Determine the Spring Constant

Trial	Mass (kg)	Weight (N)	Position of Top Of Spring (m)	Equilibrium Position (m):	Position Final (m)	Stretch Length (m)	Displacement:	Spring Constant (N/m):
1	0.050	0.490	0.840	0.569	0.545	0.295	0.024	20.41666667
2	0.075	0.735	0.840	0.569	0.540	0.300	0.029	25.34482759
3	0.100	0.980	0.840	0.569	0.530	0.310	0.039	25.12820513
4	0.150	1.470	0.840	0.569	0.510	0.330	0.059	24.91525424
5	0.175	1.715	0.840	0.569	0.505	0.335	0.064	26.796875
6	0.200	1.960	0.840	0.569	0.500	0.340	0.069	28.4057971
7	0.250	2.450	0.840	0.569	0.480	0.360	0.089	27.52808989
8	0.275	2.695	0.840	0.569	0.470	0.370	0.099	27.22222222
9	0.300	2.940	0.840	0.569	0.460	0.380	0.109	26.97247706
10	0.350	3.430	0.840	0.569	0.440	0.400	0.129	26.58914729
11	0.375	3.675	0.840	0.569	0.430	0.410	0.139	26.43884892
12	0.400	3.920	0.840	0.569	0.420	0.420	0.149	26.30872483
13	0.450	4.410	0.840	0.569	0.405	0.435	0.164	26.8902439
14	0.475	4.655	0.840	0.569	0.375	0.465	0.194	23.99484536
15	0.500	4.900	0.840	0.569	0.385	0.455	0.184	26.63043478
16	0.550	5.390	0.840	0.569	0.370	0.470	0.199	27.08542714
17	0.600	5.880	0.840	0.569	0.355	0.485	0.214	27.47663551
18	0.650	6.370	0.840	0.569	0.335	0.505	0.234	27.22222222
19	0.700	6.860	0.840	0.569	0.310	0.530	0.259	26.48648649
20	0.750	7.350	0.840	0.569	0.295	0.545	0.274	26.82481752
21	0.800	7.840	0.840	0.569	0.275	0.565	0.294	26.66666667
22	0.850	8.330	0.840	0.569	0.255	0.585	0.314	26.52866242
23	0.900	8.820	0.840	0.569	0.225	0.615	0.344	25.63953488
24	0.950	9.310	0.840	0.569	0.225	0.615	0.344	27.06395349
Average	N/A	N/A	N/A	N/A	N/A	N/A	N/A	26.27404443

16.3 Appendix C: Oscillation Data from Vernier Motion Detector 2, Calculated Sinunsoidal Regression for Position and Calculated Instantaneous Velocities

Time	Position	Velocity	Acceleration	Regression Based Position	Instantaneous Velocity
0	0.29	-0.631	2.918	0.293019326	-0.697045175
0.03	0.267	-0.546	3.435	0.27345405	-0.602201892
0.07	0.25	-0.423	4.125	0.252606517	-0.432918046
0.1	0.238	-0.271	4.788	0.241847733	-0.281527226
0.13	0.232	-0.095	5.231	0.235846683	-0.116977023
0.17	0.231	0.086	5.369	0.235697211	0.109555432
0.2	0.237	0.268	5.246	0.241480843	0.274511069
0.23	0.249	0.447	4.711	0.25203936	0.426635277
0.27	0.268	0.593	3.818	0.272662555	0.59734586
0.3	0.289	0.706	2.767	0.292101785	0.693531034
0.33	0.316	0.778	1.656	0.313949733	0.757298594
0.37	0.342	0.812	0.583	0.345051818	0.786979523
0.4	0.371	0.824	-0.777	0.368443829	0.76638476
0.43	0.398	0.786	-2.813	0.390676113	0.709966993
0.47	0.427	0.642	-4.872	0.416732921	0.583803452
0.5	0.444	0.416	-5.599	0.43240192	0.456708337
0.53	0.453	0.23	-5.139	0.443921608	0.308265376
0.57	0.458	0.08	-4.701	0.451927251	0.089230117
0.6	0.458	-0.071	-4.625	0.452052278	-0.080927614
0.63	0.453	-0.219	-4.881	0.447109481	-0.247302557
0.67	0.444	-0.39	-5.222	0.433068055	-0.449881173
0.7	0.428	-0.588	-4.828	0.417586704	-0.578169687
0.73	0.404	-0.753	-3.226	0.39864855	-0.679432906
0.77	0.375	-0.813	-1.24	0.369568634	-0.764440617
0.8	0.348	-0.813	0.235	0.346208204	-0.786826512
0.83	0.32	-0.794	1.524	0.322727405	-0.772433897
0.87	0.294	-0.714	2.765	0.293123829	-0.697440304
0.9	0.272	-0.596	3.627	0.27354435	-0.602749976
0.93	0.254	-0.469	4.345	0.257241015	-0.479885398
0.97	0.239	-0.31	5.05	0.241889988	-0.282322495
1	0.232	-0.122	5.416	0.235864279	-0.117819286
1.03	0.232	0.063	5.394	0.234875989	0.052191132
1.07	0.237	0.243	5.136	0.241439759	0.273712607
1.1	0.248	0.41	4.665	0.251975469	0.425919239
1.13	0.264	0.565	3.837	0.266795514	0.558217195
1.17	0.287	0.677	2.642	0.291997868	0.69312792
1.2	0.311	0.738	1.436	0.313836245	0.757066181
1.23	0.336	0.774	0.163	0.33706741	0.785617014
1.27	0.364	0.756	-1.268	0.368328948	0.766578203
1.3	0.388	0.684	-2.531	0.390569674	0.71033421
1.33	0.41	0.58	-3.555	0.410616449	0.620887176
1.37	0.428	0.442	-4.341	0.432333416	0.457401783
1.4	0.44	0.282	-4.823	0.443875346	0.309048933
1.43	0.447	0.113	-5.035	0.450731667	0.146250257
1.47	0.448	-0.059	-5.033	0.452064344	-0.080080289
1.5	0.443	-0.228	-4.835	0.447146487	-0.246493763
1.53	0.433	-0.386	-4.429	0.437390117	-0.401385416
1.57	0.417	-0.528	-3.802	0.417673318	-0.577591414
1.6	0.397	-0.645	-2.935	0.398750352	-0.679002572
1.63	0.373	-0.735	-1.715	0.377251046	-0.748675217
1.67	0.346	-0.768	-0.255	0.346326133	-0.786805927
1.7	0.321	-0.743	1.008	0.32284319	-0.772596819
1.73	0.296	-0.698	2.183	0.300332024	-0.722274338
1.77	0.273	-0.602	3.401	0.273634732	-0.603297353
1.8	0.255	-0.462	4.313	0.257312991	-0.480560256
1.83	0.241	-0.305	4.904	0.245026096	-0.335360405
1.87	0.234	-0.13	5.276	0.235882001	-0.118661411
1.9	0.233	0.052	5.419	0.23486823	0.051341185
1.93	0.237	0.242	5.205	0.238938436	0.218943947
1.97	0.249	0.41	4.621	0.251911685	0.425202702

2	0.265	0.549	3.958	0.266711893	0.557616396
2.03	0.286	0.677	3.144	0.285107615	0.663965514
2.07	0.311	0.771	1.966	0.313722793	0.756832881
2.1	0.339	0.806	0.723	0.336949664	0.785565245
2.13	0.365	0.824	-0.741	0.360488936	0.777578055
2.17	0.395	0.783	-2.841	0.390463181	0.710700596
2.2	0.421	0.628	-4.651	0.410523351	0.621410276
2.23	0.439	0.427	-5.128	0.427456878	0.503073471
2.27	0.448	0.259	-4.722	0.443828967	0.309832128
2.3	0.455	0.122	-4.461	0.450709684	0.147087128
2.33	0.457	-0.026	-4.59	0.452585334	-0.022533143
2.37	0.453	-0.174	-5.036	0.447183371	-0.245684679
2.4	0.446	-0.353	-5.588	0.437450222	-0.400652487
2.43	0.431	-0.58	-5.092	0.423331792	-0.536892657
2.47	0.405	-0.739	-3.226	0.398852089	-0.678571442
2.5	0.379	-0.796	-1.21	0.377363239	-0.74841206
2.53	0.351	-0.796	0.199	0.354297743	-0.78326977
2.57	0.326	-0.772	1.241	0.322959	-0.772758835
2.6	0.299	-0.716	2.298	0.300440305	-0.722612305
2.63	0.277	-0.621	3.374	0.279940561	-0.638688819
2.67	0.256	-0.489	4.338	0.257385069	-0.48123455
2.7	0.243	-0.321	4.96	0.245076418	-0.336130804
2.73	0.235	-0.148	5.222	0.237374584	-0.175315348
2.77	0.233	0.033	5.222	0.234860599	0.050491177
2.8	0.237	0.207	4.97	0.238905682	0.218125649
2.83	0.247	0.368	4.56	0.24784602	0.375564307
2.87	0.262	0.514	3.997	0.266628362	0.557014945
2.9	0.282	0.645	3.085	0.285008133	0.663507759
2.93	0.306	0.731	1.798	0.306128199	0.738986333
2.97	0.332	0.767	0.393	0.336831927	0.785512555
3	0.359	0.755	-0.986	0.360372382	0.777709325
3.03	0.383	0.695	-2.185	0.383130393	0.733553749
3.07	0.406	0.612	-3.359	0.410430174	0.621932647
3.1	0.426	0.467	-4.308	0.427381427	0.503728245
3.13	0.437	0.301	-4.562	0.440418043	0.361978151
3.17	0.445	0.155	-4.565	0.450687576	0.147923826
3.2	0.448	0.007	-4.756	0.452588648	-0.021681687
3.23	0.446	-0.161	-4.93	0.449396825	-0.190273737
3.27	0.437	-0.202	-4.723	0.437510218	-0.399919087
3.3	0.423	-0.486	-4.135	0.423412215	-0.536269511
3.33	0.404	-0.615	-3.287	0.405585107	-0.647553172
3.37	0.382	-0.718	-2.048	0.377475392	-0.748148026
3.4	0.355	-0.76	-0.568	0.354415134	-0.783185968
3.43	0.329	-0.745	0.678	0.330850891	-0.78161557
3.47	0.305	-0.713	1.835	0.300548636	-0.722949425
3.5	0.28	-0.629	3.063	0.280036325	-0.63918619
3.53	0.262	-0.497	3.965	0.262496707	-0.525545575
3.57	0.247	-0.358	4.654	0.245126855	-0.33690081
3.6	0.237	-0.189	5.302	0.237400923	-0.176145636
3.63	0.234	0.006	5.574	0.234640582	-0.007156915
3.67	0.237	0.201	5.251	0.23887305	0.217307095
3.7	0.248	0.368	4.53	0.247789786	0.374815529
3.73	0.263	0.5	3.82	0.261186508	0.514804018
3.77	0.282	0.619	3.145	0.28490872	0.663049227
3.8	0.304	0.719	2.223	0.306017461	0.738692807
3.83	0.331	0.773	1.076	0.328884461	0.779807783
3.87	0.357	0.791	-0.231	0.360255808	0.777839685
3.9	0.384	0.779	-2.126	0.383020424	0.733861984
3.93	0.412	0.659	-4.174	0.403943962	0.655581486
3.97	0.43	0.462	-5.103	0.427305879	0.504382429
4	0.442	0.29	-5.101	0.440363732	0.362734297
4.03	0.45	0.121	-4.907	0.448900119	0.204130932
4.07	0.45	0.071	-4.703	0.452591834	-0.020830206
4.1	0.447	-0.185	-4.685	0.449425282	-0.189447101
4.13	0.438	-0.348	-4.618	0.441313699	-0.349208699
4.17	0.424	-0.511	-4.004	0.423492545	-0.535645737

4.2	0.403	-0.632	-2.882	0.405682127	-0.647068649
4.23	0.38	-0.699	-1.785	0.384971358	-0.728245731
4.27	0.356	-0.749	-0.687	0.354532513	-0.78310125
4.3	0.329	-0.752	0.613	0.330968048	-0.781715003
4.33	0.304	-0.709	1.915	0.307995581	-0.743789172
4.37	0.28	-0.622	3.112	0.280132165	-0.639682813
4.4	0.262	-0.492	4	0.262575523	-0.526179333
4.43	0.247	-0.346	4.573	0.248807741	-0.388080734
4.47	0.238	-0.184	4.978	0.237427386	-0.176975718
4.5	0.235	-0.01	5.21	0.234641718	-0.008008667
4.53	0.237	0.168	5.242	0.236950615	0.161332732
4.57	0.246	0.35	4.885	0.247733664	0.374066312
4.6	0.262	0.505	4.138	0.261109397	0.514159412
4.63	0.28	0.631	3.204	0.27834252	0.630219239
4.67	0.305	0.72	2.187	0.305906767	0.738398415
4.7	0.329	0.775	1.152	0.328767591	0.779692069
4.73	0.357	0.805	-0.16	0.352323269	0.784540697
4.77	0.384	0.777	-1.849	0.382910409	0.734169359
4.8	0.41	0.688	-3.657	0.403845667	0.656052421
4.83	0.433	0.522	-5.039	0.421966416	0.547269727
4.87	0.446	0.315	-5.301	0.440309309	0.363490018
4.9	0.452	0.146	-4.837	0.448869461	0.204953456
4.93	0.455	0.005	-4.61	0.452510565	0.036836787
4.97	0.453	-0.15	-4.7	0.449453614	-0.188620242
5	0.445	-0.311	-4.747	0.441365982	-0.348445137
5.03	0.432	-0.472	-4.544	0.428710034	-0.491982716
5.07	0.414	-0.629	-3.8	0.405779074	-0.646583367
5.1	0.389	-0.746	-2.404	0.385080485	-0.727922253
5.13	0.362	-0.797	-0.727	0.362444524	-0.775235981
5.17	0.335	-0.791	0.877	0.33108522	-0.781813519
5.2	0.308	-0.729	2.185	0.308107082	-0.744067237
5.23	0.285	-0.631	3.057	0.286789529	-0.671541136
5.27	0.266	-0.524	3.811	0.262654435	-0.526812475
5.3	0.249	-0.381	4.59	0.248865963	-0.38882155
5.33	0.239	-0.208	5.055	0.239507173	-0.232655999
5.37	0.235	-0.033	5.163	0.234642982	-0.00886041
5.4	0.237	0.141	5.05	0.236926497	0.160498933
5.43	0.245	0.308	4.723	0.244197779	0.3223561
5.47	0.258	0.461	4.192	0.261032382	0.513514203
5.5	0.276	0.597	3.331	0.2782481	0.629708636
5.53	0.299	0.692	2.156	0.298520096	0.716468696
5.57	0.323	0.745	0.823	0.328650739	0.779575441
5.6	0.35	0.748	-0.572	0.352205676	0.78460815
5.63	0.374	0.7	-1.784	0.375359905	0.752966042
5.67	0.397	0.632	-2.99	0.403747302	0.656522588
5.7	0.419	0.499	-4.052	0.421884345	0.547881568
5.73	0.431	0.34	-4.484	0.436363699	0.413631004
5.77	0.441	0.193	-4.652	0.448838681	0.20577574
5.8	0.444	0.032	-4.788	0.45250498	0.037687624
5.83	0.443	-0.13	-4.767	0.451082295	-0.132162119
5.87	0.436	-0.292	-4.528	0.44141815	-0.347681168
5.9	0.423	-0.441	-3.983	0.428783723	-0.491317566
5.93	0.406	-0.566	-3.151	0.412169112	-0.611988383
5.97	0.384	-0.655	-2.152	0.385189562	-0.727597922
6	0.361	-0.708	-1.108	0.362560706	-0.775088519
6.03	0.337	-0.735	0.133	0.339047117	-0.786349274
6.07	0.31	-0.707	1.571	0.308218625	-0.744344431
6.1	0.288	-0.622	2.766	0.286890214	-0.671984964
6.13	0.268	-0.512	3.614	0.268214124	-0.568215009
6.17	0.253	-0.377	4.266	0.248924296	-0.38956191
6.2	0.243	-0.228	4.85	0.239542105	-0.233469588
6.23	0.237	-0.048	5.211	0.235025422	-0.066464233
6.27	0.24	0.136	5.082	0.236902503	0.159664946
6.3	0.247	0.296	4.719	0.244149522	0.321578842
6.33	0.259	0.451	4.224	0.256046683	0.468461228
6.37	0.277	0.591	3.33	0.278153757	0.629197295

6.4	0.3	0.683	2.148	0.298412737	0.716115726
6.43	0.324	0.725	1.119	0.320785443	0.769560871
6.47	0.349	0.757	0.039	0.352088073	0.784674685
6.5	0.375	0.747	-1.518	0.375247031	0.753213541
6.53	0.401	0.658	-3.118	0.396928261	0.686545054
6.57	0.421	0.519	-4.169	0.421802182	0.548492767
6.6	0.436	0.367	-4.747	0.436301649	0.414355435
6.63	0.446	0.2	-5.097	0.446469523	0.260849952
6.67	0.45	0.016	-5.067	0.452499267	0.038538416
6.7	0.446	-0.149	-4.699	0.45110204	-0.131322345
6.73	0.439	-0.292	-4.403	0.444681407	-0.295044726
6.77	0.427	-0.44	-4.103	0.428857313	-0.490651841
6.8	0.41	-0.582	-3.322	0.412260797	-0.611452428
6.83	0.386	-0.676	-2.104	0.392456425	-0.703671989
6.87	0.363	-0.715	-0.992	0.362676866	-0.774940149
6.9	0.339	-0.74	0.106	0.339164979	-0.786384651
6.93	0.312	-0.716	1.393	0.315861942	-0.761071298
6.97	0.29	-0.646	2.628	0.286990965	-0.672428006
7	0.268	-0.537	3.699	0.268299333	-0.568804062
7.03	0.253	-0.389	4.418	0.253129013	-0.438592598
7.07	0.242	-0.233	4.801	0.239577159	-0.234282903
7.1	0.237	-0.065	4.985	0.235035447	-0.067312944
7.13	0.238	0.105	4.959	0.235569896	0.102803414
7.17	0.244	0.271	4.728	0.244101382	0.320801208
7.2	0.256	0.425	4.305	0.255976521	0.467776482
7.23	0.273	0.567	3.555	0.271948976	0.592886552
7.27	0.295	0.671	2.486	0.298305431	0.715761918
7.3	0.318	0.736	1.294	0.320670113	0.76938187
7.33	0.345	0.757	0.066	0.344108149	0.787038725
7.37	0.37	0.739	-1.135	0.37513412	0.753460158
7.4	0.395	0.688	-2.48	0.396825329	0.686961145
7.43	0.418	0.571	-3.708	0.41603018	0.588351614
7.47	0.434	0.427	-4.496	0.436239491	0.415079381
7.5	0.447	0.265	-5.013	0.446430366	0.261653448
7.53	0.452	0.083	-5.178	0.451816202	0.095997089
7.57	0.452	-0.089	-5.069	0.45112166	-0.130482417
7.6	0.446	-0.255	-4.886	0.444725569	-0.294254879
7.63	0.435	-0.414	-4.6	0.433604126	-0.44427303
7.67	0.419	-0.575	-3.881	0.412352402	-0.610915756
7.7	0.395	-0.695	-2.524	0.392561864	-0.703290031
7.73	0.37	-0.743	-1.097	0.370484253	-0.762790529
7.77	0.345	-0.758	0.111	0.339282846	-0.786419108
7.8	0.319	-0.742	1.428	0.315976028	-0.761287879
7.83	0.294	-0.663	2.697	0.293961978	-0.700571888
7.87	0.274	-0.546	3.539	0.26838463	-0.569392448
7.9	0.258	-0.423	4.196	0.253194803	-0.439299614
7.93	0.244	-0.27	4.835	0.242232316	-0.288672666
7.97	0.239	-0.09	5.127	0.235045599	-0.068161575
8	0.239	0.084	5.052	0.235554551	0.10195886
8.03	0.245	0.251	4.782	0.241115398	0.267313448
8.07	0.256	0.405	4.35	0.255906461	0.467091187
8.1	0.272	0.551	3.585	0.271860155	0.592326006
8.13	0.294	0.655	2.461	0.291168718	0.689873823
8.17	0.316	0.718	1.219	0.320554811	0.769201969
8.2	0.343	0.736	-0.024	0.343990186	0.787041977
8.23	0.367	0.708	-1.092	0.367408867	0.768093405
8.27	0.39	0.665	-2.218	0.396722335	0.687376431
8.3	0.412	0.575	-3.657	0.415941955	0.58891704
8.33	0.431	0.409	-4.685	0.43178165	0.462929991
8.37	0.44	0.234	-4.87	0.446391089	0.262456638
8.4	0.445	0.079	-4.81	0.45180175	0.096842465
8.43	0.445	-0.081	-4.822	0.452156299	-0.073298401
8.47	0.44	-0.244	-4.736	0.444769613	-0.293464686
8.5	0.429	-0.404	-4.37	0.433670662	-0.443569662
8.53	0.413	-0.552	-3.495	0.418363096	-0.57294093
8.57	0.391	-0.649	-2.266	0.392667245	-0.702907249

8.6	0.368	-0.694	-1.208	0.370598566	-0.762580245
8.63	0.345	-0.728	-0.164	0.347269441	-0.78660807
8.67	0.318	-0.714	1.102	0.316090147	-0.761503568
8.7	0.296	-0.654	2.344	0.29406701	-0.700959649
8.73	0.274	-0.554	3.452	0.27436073	-0.607650882
8.77	0.258	-0.414	4.237	0.253260698	-0.440006116
8.8	0.246	-0.263	4.719	0.242275642	-0.289464926
8.83	0.24	-0.096	5.036	0.236028319	-0.125393322
8.87	0.239	0.077	5.163	0.235539332	0.101114186
8.9	0.245	0.259	4.926	0.241075393	0.266512134
8.93	0.257	0.416	4.33	0.25140529	0.419452547
8.97	0.273	0.551	3.562	0.271771419	0.591764767
9	0.295	0.656	2.665	0.29106535	0.68946342
9.03	0.317	0.729	1.707	0.312816444	0.75493459
9.07	0.344	0.778	0.475	0.343872223	0.787044307
9.1	0.371	0.771	-1.089	0.36729373	0.768278758
9.13	0.397	0.711	-2.797	0.38960927	0.713601675
9.17	0.42	0.577	-4.23	0.415853645	0.589481776
9.2	0.436	0.407	-4.97	0.431712214	0.463618586
9.23	0.447	0.231	-5.172	0.443453712	0.316084544
9.27	0.451	0.057	-5.114	0.451787172	0.097687727
9.3	0.451	-0.114	-4.901	0.452167221	-0.072450268
9.33	0.443	-0.273	-4.545	0.447474075	-0.239201732
9.37	0.433	-0.42	-4.035	0.433737092	-0.442865774
9.4	0.415	-0.549	-3.3	0.418448925	-0.572356592
9.43	0.395	-0.647	-2.346	0.399663651	-0.675093835
9.47	0.371	-0.709	-1.241	0.370712846	-0.762369067
9.5	0.347	-0.732	-0.054	0.347387337	-0.786579192
9.53	0.321	-0.714	1.171	0.32388634	-0.774022367
9.57	0.298	-0.647	2.21	0.2941721	-0.701346588
9.6	0.278	-0.564	3.139	0.274451846	-0.608191877
9.63	0.259	-0.441	4.076	0.257965319	-0.486608547
9.67	0.247	-0.281	4.669	0.242319087	-0.290256846
9.7	0.241	-0.118	4.889	0.236047176	-0.12623416
9.73	0.239	0.049	4.915	0.234804135	0.04368907
9.77	0.244	0.214	4.767	0.241035508	0.265710508
9.8	0.254	0.371	4.421	0.251342476	0.418731558
9.83	0.269	0.519	3.715	0.265963367	0.552179903
9.87	0.289	0.629	2.666	0.290962043	0.689052209
9.9	0.312	0.7	1.485	0.312703312	0.754693322
9.93	0.338	0.726	0.447	0.335890324	0.785057922
9.97	0.361	0.712	-0.105	0.367178566	0.768463213
10	0.384	0.7	-0.31	0.389502288	0.713960553

Table 4: Oscillation Data Returned from Vernier Motion Detector 2

16.4 Appendix D: Energy in Components Data (Microsoft Excel 2013)

Time	Total Energy (J)	Kinetic Energy (J)	Spring Potential Energy (J)	Gravitational Potential Energy (J)
0	2.285746438	0.121970062	0.736902909	1.426873467
0.03	2.292209442	0.091036515	0.887465287	1.31370764
0.07	2.28485095	0.047048174	1.007739443	1.230063333
0.1	2.288155646	0.019896294	1.097239059	1.171020293
0.13	2.288350338	0.003435046	1.143416519	1.141498773
0.17	2.290796816	0.003013001	1.151205295	1.13657852
0.2	2.289886197	0.01891695	1.104869207	1.16610004
0.23	2.285887915	0.0456925	1.015052335	1.22514308
0.27	2.288830407	0.089574235	0.880628279	1.318627893
0.3	2.285854837	0.120743342	0.743158282	1.421953213
0.33	2.282310928	0.143967908	0.583542967	1.554800053
0.37	2.286255926	0.155474174	0.448055112	1.68272664
0.4	2.290875659	0.147443324	0.318018348	1.825413987
0.43	2.30173173	0.126534138	0.216936765	1.958260827
0.47	2.316342292	0.085558805	0.129835314	2.100948173
0.5	2.326066742	0.052361162	0.0891131	2.18459248
0.53	2.323377368	0.023855081	0.070647527	2.22887476
0.57	2.316788963	0.001998731	0.061314205	2.253476027
0.6	2.316434319	0.001644087	0.061314205	2.253476027
0.63	2.314875123	0.015352836	0.070647527	2.22887476
0.67	2.324512987	0.050807407	0.0891131	2.18459248
0.7	2.317012282	0.08391547	0.127228385	2.105868427
0.73	2.300758185	0.115884285	0.197091553	1.987782347
0.77	2.293618364	0.146696213	0.301827151	1.845095
0.8	2.286988615	0.155413722	0.419326732	1.71224816
0.83	2.28579655	0.149780074	0.561535409	1.574481067
0.87	2.280808662	0.122108382	0.7121458	1.44655448
0.9	2.283055833	0.091202301	0.853544625	1.338308907
0.93	2.286307065	0.057810465	0.978752253	1.249744347
0.97	2.285584756	0.020008861	1.089635348	1.175940547
1	2.288399982	0.00348469	1.143416519	1.141498773
1.03	2.285599085	0.000683793	1.143416519	1.141498773
1.07	2.289776311	0.018807064	1.104869207	1.16610004
1.1	2.288153746	0.045539254	1.022391665	1.220222827
1.13	2.285305423	0.078223602	0.90813494	1.29894688
1.17	2.288464066	0.120603019	0.75574834	1.412112707
1.2	2.28572561	0.143879555	0.611647268	1.530198787
1.23	2.28587667	0.15493629	0.47773526	1.65320512
1.27	2.285860786	0.147517766	0.347370807	1.790972213
1.3	2.287850519	0.126665066	0.252127159	1.909058293
1.33	2.292275548	0.096773572	0.178198109	2.017303867
1.37	2.2856171	0.052520288	0.127228385	2.105868427
1.4	2.286895382	0.023976506	0.098007409	2.164911467
1.43	2.287442594	0.005369386	0.082719967	2.19935324
1.47	2.286525132	0.00160984	0.080641799	2.204273493
1.5	2.286221825	0.015252578	0.09129702	2.179672227
1.53	2.285504051	0.040444044	0.114590314	2.130469693
1.57	2.29285202	0.083747693	0.157358687	2.05174564
1.6	2.289414943	0.115737536	0.220336834	1.953340573
1.63	2.285832211	0.140707844	0.309869874	1.835254493
1.67	2.286610351	0.155405591	0.428797107	1.702407653
1.7	2.285344199	0.149843264	0.556099615	1.57940132
1.73	2.287279984	0.130959124	0.699925873	1.456394987
1.77	2.281436991	0.091368024	0.846839807	1.34322916
1.8	2.284209327	0.057973176	0.971571551	1.2546646
1.83	2.288521161	0.028232866	1.074507242	1.185781053
1.87	2.282792243	0.003534682	1.12791828	1.15133928
1.9	2.28273491	0.000661703	1.13565418	1.146419027
1.93	2.283002894	0.012033647	1.104869207	1.16610004
1.97	2.285581573	0.045386158	1.015052335	1.22514308
2	2.283141063	0.078055312	0.901218618	1.303867133
2.03	2.279943576	0.110668096	0.762083027	1.407192453
2.07	2.285636947	0.143790891	0.611647268	1.530198787
2.1	2.285657967	0.154915872	0.462776215	1.66796588

2.13	2.290772441	0.15178169	0.343098284	1.795892467
2.17	2.297512117	0.126795766	0.227216284	1.943500067
2.2	2.31439544	0.096936705	0.146032082	2.071426653
2.23	2.323820543	0.063532248	0.100297082	2.159991213
2.27	2.309013475	0.024098183	0.080641799	2.204273493
2.3	2.310981163	0.005431012	0.066834884	2.238715267
2.33	2.311811227	0.00012746	0.063127994	2.248555773
2.37	2.3146749	0.015152613	0.070647527	2.22887476
2.4	2.319554037	0.040296477	0.084824573	2.194432987
2.43	2.312556709	0.072361293	0.119566228	2.120629187
2.47	2.302169759	0.115590608	0.193876551	1.9927026
2.5	2.291443919	0.140608944	0.286058962	1.864776013
2.53	2.28634022	0.154011845	0.405319455	1.72700892
2.57	2.283225916	0.149906116	0.529317213	1.604002587
2.6	2.284031725	0.13108171	0.681794268	1.471155747
2.63	2.28559746	0.102402373	0.820284914	1.362910173
2.67	2.282138119	0.058135979	0.964417287	1.259584853
2.7	2.283469176	0.028362729	1.059484887	1.19562156
2.73	2.284183979	0.007715628	1.120208818	1.156259533
2.77	2.282713181	0.000639974	1.13565418	1.146419027
2.8	2.282913111	0.011943864	1.104869207	1.16610004
2.83	2.280467894	0.035407887	1.029757434	1.215302573
2.87	2.289040292	0.07788702	0.922046899	1.289106373
2.9	2.285713146	0.110515554	0.787686152	1.38751144
2.93	2.283099544	0.137089504	0.64041252	1.50559752
2.97	2.286469983	0.154895091	0.498050785	1.633524107
3	2.287333876	0.151832941	0.369129988	1.766370947
3.03	2.29025212	0.135081313	0.27071378	1.884457027
3.07	2.285410588	0.097099748	0.190687987	1.997622853
3.1	2.292194336	0.063697736	0.13246868	2.09602792
3.13	2.287998888	0.032892441	0.10495574	2.150150707
3.17	2.281961326	0.005492975	0.086955618	2.189512733
3.2	2.285033302	0.00011801	0.080641799	2.204273493
3.23	2.288345995	0.009088435	0.084824573	2.194432987
3.27	2.295255533	0.040149086	0.10495574	2.150150707
3.3	2.293987985	0.072193418	0.140527407	2.08126716
3.33	2.29013848	0.10526458	0.197091553	1.987782347
3.37	2.294556942	0.14050975	0.274510419	1.879536773
3.4	2.287682043	0.153978892	0.387013218	1.746689933
3.43	2.285690387	0.153362012	0.513565028	1.618763347
3.47	2.278126197	0.131204045	0.646244884	1.500677267
3.5	2.2808792	0.102561924	0.800646343	1.377670933
3.53	2.280488215	0.069334943	0.922046899	1.289106373
3.57	2.273552831	0.028492824	1.029757434	1.215302573
3.6	2.27875813	0.007788883	1.104869207	1.16610004
3.63	2.279270418	1.28583E-05	1.12791828	1.15133928
3.67	2.282823637	0.01185439	1.104869207	1.16610004
3.7	2.277881332	0.03526684	1.022391665	1.220222827
3.73	2.275633979	0.066529652	0.9150777	1.294026627
3.77	2.28556045	0.110362858	0.787686152	1.38751144
3.8	2.284841322	0.136980622	0.652103687	1.495757013
3.83	2.28445303	0.152653415	0.503195762	1.628603853
3.87	2.286433013	0.151883846	0.378018727	1.75653044
3.9	2.291515718	0.135194858	0.26694358	1.88937728
3.93	2.307147057	0.107890885	0.172111799	2.027144373
3.97	2.301666066	0.06386329	0.122093843	2.115708933
4	2.301289357	0.033030005	0.093507379	2.174751973
4.03	2.301139194	0.010460418	0.076564776	2.214114
4.07	2.290787699	0.000108923	0.076564776	2.214114
4.1	2.291082845	0.009009638	0.082719967	2.19935324
4.13	2.288296842	0.03061269	0.102613192	2.15507096
4.17	2.296027709	0.072025569	0.137814727	2.086187413
4.2	2.2883022	0.105107113	0.200332994	1.982862093
4.23	2.285012757	0.133133481	0.282183009	1.869696267
4.27	2.288058521	0.153945581	0.382502753	1.751610187
4.3	2.285729409	0.153401034	0.513565028	1.618763347

4.33	2.286737946	0.138877246	0.652103687	1.495757013
4.37	2.281038635	0.102721359	0.800646343	1.377670933
4.4	2.280655538	0.069502266	0.922046899	1.289106373
4.43	2.282867298	0.037807291	1.029757434	1.215302573
4.47	2.276121818	0.007862466	1.097239059	1.171020293
4.5	2.276484452	1.6101E-05	1.120208818	1.156259533
4.53	2.277503205	0.006533958	1.104869207	1.16610004
4.57	2.282657951	0.035125991	1.03714964	1.21038232
4.6	2.277516419	0.066363147	0.922046899	1.289106373
4.63	2.278021764	0.099704488	0.800646343	1.377670933
4.67	2.283793613	0.136871461	0.646244884	1.500677267
4.7	2.284936489	0.152608114	0.513565028	1.618763347
4.73	2.289061214	0.154512047	0.378018727	1.75653044
4.77	2.291628994	0.135308133	0.26694358	1.88937728
4.8	2.303547923	0.108045946	0.178198109	2.017303867
4.83	2.320245533	0.075185526	0.114590314	2.130469693
4.87	2.312425337	0.033167777	0.084824573	2.194432987
4.9	2.307092898	0.010544886	0.072593505	2.223954507
4.93	2.30589079	0.000340639	0.066834884	2.238715267
4.97	2.30845345	0.008931162	0.070647527	2.22887476
5	2.306947316	0.030478965	0.086955618	2.189512733
5.03	2.303376355	0.060761863	0.117065052	2.12554944
5.07	2.308065638	0.104949518	0.16613124	2.03698488
5.1	2.29548293	0.133015235	0.248489148	1.913978547
5.13	2.287995603	0.150868731	0.355995165	1.781131707
5.17	2.284499053	0.153439702	0.482774484	1.648284867
5.2	2.283246236	0.138981104	0.628827105	1.515438027
5.23	2.283924225	0.113207874	0.768444151	1.4022722
5.27	2.272785748	0.069669628	0.894328733	1.308787387
5.3	2.278147186	0.037951771	1.015052335	1.22514308
5.33	2.279164032	0.013588137	1.089635348	1.175940547
5.37	2.276488059	1.97078E-05	1.120208818	1.156259533
5.4	2.277435843	0.006466595	1.104869207	1.16610004
5.43	2.276116092	0.026085741	1.044568284	1.205462067
5.47	2.285810128	0.066196696	0.950188072	1.26942536
5.5	2.284416892	0.099542992	0.82688398	1.35798992
5.53	2.281812301	0.128862286	0.681794268	1.471155747
5.57	2.28711163	0.152562463	0.54530734	1.589241827
5.6	2.286589394	0.154538618	0.409962109	1.722088667
5.63	2.288335362	0.142325322	0.305835293	1.840174747
5.67	2.281878273	0.108200866	0.220336834	1.953340573
5.7	2.288582388	0.075353733	0.151642508	2.061586147
5.73	2.283144861	0.042949446	0.119566228	2.120629187
5.77	2.276205564	0.010629669	0.095744175	2.16983172
5.8	2.274062137	0.000356557	0.0891131	2.18459248
5.83	2.275354003	0.004384755	0.09129702	2.179672227
5.87	2.28290064	0.03034546	0.107324727	2.145230453
5.9	2.282392244	0.060597677	0.140527407	2.08126716
5.93	2.2823303	0.094019459	0.190687987	1.997622853
5.97	2.289217589	0.132896729	0.26694358	1.88937728
6	2.287369796	0.150811341	0.360347002	1.776211453
6.03	2.286073099	0.155225252	0.472722474	1.658125373
6.07	2.281710651	0.139084675	0.617347443	1.525278533
6.1	2.279830616	0.113357564	0.749440092	1.41703296
6.13	2.280306877	0.081050705	0.880628279	1.318627893
6.17	2.268879925	0.038096437	0.985959394	1.244824093
6.2	2.268789784	0.013683337	1.059484887	1.19562156
6.23	2.272078185	0.001108938	1.104869207	1.16610004
6.27	2.269318442	0.006399566	1.082058076	1.1808608
6.3	2.271020105	0.025960098	1.029757434	1.215302573
6.33	2.272549487	0.055090752	0.943113122	1.274345613
6.37	2.282576482	0.099381395	0.820284914	1.362910173
6.4	2.280614625	0.128735349	0.675803276	1.476076
6.43	2.282780888	0.148667948	0.53995086	1.59416208
6.47	2.286364443	0.154564828	0.414631202	1.717168413
6.5	2.289341052	0.142418901	0.301827151	1.845095

6.53	2.298239859	0.118323084	0.206895188	1.973021587
6.57	2.292980686	0.075521951	0.146032082	2.071426653
6.6	2.2956552	0.04310002	0.107324727	2.145230453
6.63	2.296338545	0.017080985	0.084824573	2.194432987
6.67	2.291051613	0.000372837	0.076564776	2.214114
6.7	2.28358677	0.00432921	0.084824573	2.194432987
6.73	2.282141096	0.021852801	0.100297082	2.159991213
6.77	2.291217058	0.060433571	0.129835314	2.100948173
6.8	2.28935683	0.093854854	0.178198109	2.017303867
6.83	2.283000506	0.124300226	0.259482493	1.899217787
6.87	2.288475336	0.150753609	0.351669767	1.78605196
6.9	2.285981314	0.155239219	0.462776215	1.66796588
6.93	2.28649849	0.145405917	0.605973532	1.53511904
6.97	2.277283463	0.113507087	0.736902909	1.426873467
7	2.28047501	0.081218838	0.880628279	1.318627893
7.03	2.279073129	0.048289642	0.985959394	1.244824093
7.07	2.27146299	0.013778838	1.066982845	1.190701307
7.1	2.272106687	0.00113744	1.104869207	1.16610004
7.13	2.270912408	0.002653056	1.097239059	1.171020293
7.17	2.278389878	0.025834698	1.052013367	1.200541813
7.2	2.278931958	0.054929818	0.964417287	1.259584853
7.23	2.278310815	0.088241847	0.846839807	1.34322916
7.27	2.286105524	0.128608173	0.706022618	1.451474733
7.3	2.285725668	0.148598796	0.572486312	1.56464056
7.33	2.286556917	0.155497566	0.433571951	1.6974874
7.37	2.285138154	0.142512178	0.322132243	1.820493733
7.4	2.289182901	0.11846655	0.227216284	1.943500067
7.43	2.298050374	0.086897102	0.154487379	2.056665893
7.47	2.290782717	0.043250757	0.112142013	2.135389947
7.5	2.299259583	0.017186376	0.082719967	2.19935324
7.53	2.298861395	0.002313383	0.072593505	2.223954507
7.57	2.300822021	0.004274008	0.072593505	2.223954507
7.6	2.300993516	0.021735956	0.084824573	2.194432987
7.63	2.29957894	0.049548589	0.109720151	2.1403102
7.67	2.306918829	0.093690174	0.151642508	2.061586147
7.7	2.294881672	0.124165321	0.227216284	1.943500067
7.73	2.288689568	0.146063592	0.322132243	1.820493733
7.77	2.286312175	0.155252824	0.433571951	1.6974874
7.8	2.282047142	0.145488686	0.566997642	1.569560813
7.83	2.281907684	0.123207404	0.7121458	1.44655448
7.87	2.269697795	0.081386955	0.840161427	1.348149413
7.9	2.268058887	0.048445455	0.950188072	1.26942536
7.93	2.273474267	0.020919087	1.052013367	1.200541813
7.97	2.266742196	0.001166301	1.089635348	1.175940547
8	2.268185539	0.002609644	1.089635348	1.175940547
8.03	2.267968309	0.017937958	1.044568284	1.205462067
8.07	2.278771131	0.054768991	0.964417287	1.259584853
8.1	2.279928602	0.08807507	0.853544625	1.338308907
8.13	2.278173543	0.119473263	0.7121458	1.44655448
8.17	2.286872331	0.148529311	0.583542967	1.554800053
8.2	2.286346698	0.155498851	0.443200954	1.687646893
8.23	2.28846703	0.148101503	0.334632554	1.805732973
8.27	2.282386201	0.118609825	0.244877576	1.9188988
8.3	2.286320376	0.087064204	0.172111799	2.027144373
8.33	2.293992907	0.053797492	0.119566228	2.120629187
8.37	2.280210927	0.017292051	0.098007409	2.164911467
8.4	2.278822658	0.002354307	0.086955618	2.189512733
8.43	2.277817067	0.001348716	0.086955618	2.189512733
8.47	2.284538249	0.021619373	0.098007409	2.164911467
8.5	2.284828399	0.049391824	0.124647895	2.11078868
8.53	2.283577458	0.082404531	0.1691083	2.032064627
8.57	2.289141693	0.124030198	0.241292442	1.923819053
8.6	2.287075642	0.14598307	0.330439345	1.810653227
8.63	2.286386792	0.155327441	0.433571951	1.6974874
8.67	2.28269801	0.145571138	0.572486312	1.56464056
8.7	2.27966469	0.12334383	0.699925873	1.456394987

8.73	2.281002286	0.092691446	0.840161427	1.348149413
8.77	2.268214836	0.048601404	0.950188072	1.26942536
8.8	2.268566029	0.021034069	1.03714964	1.21038232
8.83	2.266865995	0.003947119	1.082058076	1.1808608
8.87	2.26814248	0.002566585	1.089635348	1.175940547
8.9	2.267860927	0.017830576	1.044568284	1.205462067
8.93	2.265961482	0.044166915	0.95728946	1.264505107
8.97	2.27797721	0.087908243	0.846839807	1.34322916
9	2.276828508	0.119331157	0.706022618	1.451474733
9.03	2.28079221	0.143070483	0.57800142	1.559720307
9.07	2.286440152	0.155499772	0.438373233	1.692567147
9.1	2.291605325	0.14817299	0.318018348	1.825413987
9.13	2.301510446	0.127833039	0.220336834	1.953340573
9.17	2.302561739	0.087231263	0.148824076	2.0665064
9.2	2.306512835	0.053957655	0.107324727	2.145230453
9.23	2.307153807	0.025080599	0.082719967	2.19935324
9.27	2.295995759	0.002395584	0.074565922	2.219034253
9.3	2.294917859	0.001317684	0.074565922	2.219034253
9.33	2.285332739	0.014363492	0.09129702	2.179672227
9.37	2.294295198	0.049235191	0.114590314	2.130469693
9.4	2.28732228	0.082236529	0.163180618	2.041905133
9.43	2.285125216	0.114408865	0.227216284	1.943500067
9.47	2.289334564	0.145902229	0.318018348	1.825413987
9.5	2.286692644	0.155316037	0.4240487	1.707327907
9.53	2.285897672	0.150396737	0.556099615	1.57940132
9.57	2.277527234	0.123480042	0.687811699	1.466235493
9.6	2.274399279	0.092856567	0.813712285	1.367830427
9.63	2.276900385	0.05944165	0.943113122	1.274345613
9.67	2.266209324	0.021149317	1.029757434	1.215302573
9.7	2.264288527	0.004000232	1.074507242	1.185781053
9.73	2.266055051	0.000479156	1.089635348	1.175940547
9.77	2.270278654	0.017723474	1.052013367	1.200541813
9.8	2.27251181	0.04401521	0.978752253	1.249744347
9.83	2.273906583	0.076540727	0.873817708	1.323548147
9.87	2.284300351	0.119188856	0.743158282	1.421953213
9.9	2.284071622	0.14297905	0.605973532	1.53511904
9.93	2.285497597	0.154715845	0.467736125	1.663045627
9.97	2.284802603	0.148244148	0.360347002	1.776211453
10	2.284282509	0.127961649	0.26694358	1.88937728

Table 5: Energy in its Components (Instantaneous Velocities and Effective Mass) Complete Presentation-Microsoft Excel 2013

16.5 Appendix E: Uncertainty at Each Data Point-Equation Rearrangement Method

Time	Position	Term 1	Term 2	Term 3	Term 4	Term 5
(s)	(m)	$(\frac{v^2}{2}+\frac{g^2}{k}+hg)(\Delta m)$	$mv\Delta v$	$(\frac{v^2}{2}+\frac{mg^2}{k}+hg)(\Delta m)$	$(k(h-h_{eq})+mg)\Delta h$	$(-k(h-h_{eq})+mg)\Delta h_{eq}$
0	0.29	0.014726305	0.0015062	-0.016639082	-0.01797784	0.0026438
0.03	0.267	0.013865268	0.0015062	-0.015224582	-0.01639156	0.003859948
0.07	0.25	0.013102624	0.0015062	-0.013839082	-0.01427652	0.00475884
0.1	0.238	0.012587584	0.0015062	-0.012687082	-0.01269024	0.005393352
0.13	0.232	0.012312823	0.0015062	-0.012057082	-0.01110396	0.005710608
0.17	0.231	0.012280901	0.0015062	-0.011948582	-0.00898892	0.005763484
0.2	0.237	0.012552332	0.0015062	-0.012584582	-0.00740264	0.005446228
0.23	0.249	0.013065124	0.0015062	-0.013748582	-0.00581636	0.004811716
0.27	0.268	0.013885931	0.0015062	-0.015297082	-0.00370132	0.003807072
0.3	0.289	0.014689575	0.0015062	-0.016588582	-0.00211504	0.002696676
0.33	0.316	0.015622149	0.0015062	-0.017601082	-0.00052876	0.001269024
0.37	0.342	0.016455303	0.0015062	-0.017887082	0.00158628	-0.000105752
0.4	0.371	0.017259916	0.0015062	-0.017408582	0.00317256	-0.001639156
0.43	0.398	0.017928777	0.0015062	-0.016207082	0.00475884	-0.003066808
0.47	0.427	0.018536537	0.0015062	-0.014104582	0.00687388	-0.004600212
0.5	0.444	0.018837971	0.0015062	-0.012481082	0.00846016	-0.005499104
0.53	0.453	0.018932239	0.0015062	-0.011504582	0.01004644	-0.005974988
0.57	0.458	0.01894864	0.0015062	-0.010927082	0.01216148	-0.006239368
0.6	0.458	0.018946521	0.0015062	-0.010927082	0.01374776	-0.006239368
0.63	0.453	0.018881435	0.0015062	-0.011504582	0.01533404	-0.005974988
0.67	0.444	0.018828687	0.0015062	-0.012481082	0.01744908	-0.005499104
0.7	0.428	0.018556118	0.0015062	-0.014017082	0.01903536	-0.004653088
0.73	0.404	0.018041541	0.0015062	-0.015841082	0.02062164	-0.003384064
0.77	0.375	0.017373052	0.0015062	-0.017276582	0.02273668	-0.00185066
0.8	0.348	0.016631341	0.0015062	-0.017857082	0.02432296	-0.000423008
0.83	0.32	0.015774479	0.0015062	-0.017689082	0.02590924	0.00105752
0.87	0.294	0.014844732	0.0015062	-0.016831082	0.02802428	0.002432296
0.9	0.272	0.014013259	0.0015062	-0.015577082	0.02961056	0.003595568
0.93	0.254	0.013284532	0.0015062	-0.014191082	0.03119684	0.004547336
0.97	0.239	0.012617656	0.0015062	-0.012788582	0.03331188	0.005340476
1	0.232	0.012313119	0.0015062	-0.012057082	0.03489816	0.005710608
1.03	0.232	0.012296383	0.0015062	-0.012057082	0.03648444	0.005710608
1.07	0.237	0.012551675	0.0015062	-0.012584582	0.03859948	0.005446228
1.1	0.248	0.013034808	0.0015062	-0.013657082	0.04018576	0.004864592
1.13	0.264	0.013700507	0.0015062	-0.015001082	0.04177204	0.004018576
1.17	0.287	0.014629937	0.0015062	-0.016484582	0.04388708	0.002802428
1.2	0.311	0.015474621	0.0015062	-0.017468582	0.04547336	0.001533404
1.23	0.336	0.016275689	0.0015062	-0.017881082	0.04705964	0.000211504
1.27	0.364	0.017054561	0.0015062	-0.017601082	0.04917468	-0.001269024
1.3	0.388	0.017635559	0.0015062	-0.016737082	0.05076096	-0.002538048
1.33	0.41	0.018103749	0.0015062	-0.015439082	0.05234724	-0.00370132
1.37	0.428	0.018368522	0.0015062	-0.014017082	0.05446228	-0.004653088
1.4	0.44	0.018550764	0.0015062	-0.012889082	0.05604856	-0.0052876
1.43	0.447	0.018645381	0.0015062	-0.012164582	0.05763484	-0.005657732
1.47	0.448	0.018652317	0.0015062	-0.012057082	0.05974988	-0.005710608
1.5	0.443	0.018586836	0.0015062	-0.012584582	0.06133616	-0.005446228
1.53	0.433	0.018443363	0.0015062	-0.013564582	0.06292244	-0.004917468
1.57	0.417	0.018231715	0.0015062	-0.014924582	0.06503748	-0.004071452
1.6	0.397	0.017834864	0.0015062	-0.016264582	0.06662376	-0.003013932
1.63	0.373	0.017278469	0.0015062	-0.017344582	0.06821004	-0.001744908
1.67	0.346	0.016572493	0.0015062	-0.017871082	0.07032508	-0.000317256
1.7	0.321	0.015804256	0.0015062	-0.017708582	0.07191136	0.001004644
1.73	0.296	0.014956418	0.0015062	-0.016921082	0.07349764	0.002326544
1.77	0.273	0.014043649	0.0015062	-0.015644582	0.07561268	0.003542692
1.8	0.255	0.013314905	0.0015062	-0.014276582	0.07719896	0.00449446
1.83	0.241	0.012725597	0.0015062	-0.012988582	0.07878524	0.005234724
1.87	0.234	0.012372218	0.0015062	-0.012271082	0.08090028	0.005604856
1.9	0.233	0.012325651	0.0015062	-0.012164582	0.08248656	0.005657732
1.93	0.237	0.012511202	0.0015062	-0.012584582	0.08407284	0.005446228
1.97	0.249	0.013063293	0.0015062	-0.013748582	0.08618788	0.004811716
2	0.265	0.013728901	0.0015062	-0.015076582	0.08777416	0.0039657

2.03	0.286	0.014541173	0.0015062	-0.016431082	0.08936044	0.002855304
2.07	0.311	0.015474091	0.0015062	-0.017468582	0.09147548	0.001533404
2.1	0.339	0.016363767	0.0015062	-0.017888582	0.09306176	5.2876E-05
2.13	0.365	0.017109439	0.0015062	-0.017576582	0.09464804	-0.0013219
2.17	0.395	0.01784214	0.0015062	-0.016376582	0.09676308	-0.00290818
2.2	0.421	0.018428123	0.0015062	-0.014608582	0.09834936	-0.004282956
2.23	0.439	0.018757722	0.0015062	-0.012988582	0.09993564	-0.005234724
2.27	0.448	0.018786691	0.0015062	-0.012057082	0.10205068	-0.005710608
2.3	0.455	0.018880949	0.0015062	-0.011276582	0.10363696	-0.00608074
2.33	0.457	0.018908059	0.0015062	-0.011044582	0.10522324	-0.006186492
2.37	0.453	0.018880239	0.0015062	-0.011504582	0.10733828	-0.005974988
2.4	0.446	0.018824681	0.0015062	-0.012271082	0.10892456	-0.005604856
2.43	0.431	0.018575278	0.0015062	-0.013748582	0.11051084	-0.004811716
2.47	0.405	0.018069186	0.0015062	-0.015776582	0.11262588	-0.00343694
2.5	0.379	0.017454278	0.0015062	-0.017128582	0.11421216	-0.002062164
2.53	0.351	0.016711165	0.0015062	-0.017828582	0.11579844	-0.000581636
2.57	0.326	0.015951632	0.0015062	-0.017791082	0.11791348	0.000740264
2.6	0.299	0.01504535	0.0015062	-0.017048582	0.11949976	0.002167916
2.63	0.277	0.014227183	0.0015062	-0.015904582	0.12108604	0.003331188
2.67	0.256	0.013345277	0.0015062	-0.014361082	0.12320108	0.004441584
2.7	0.243	0.012785173	0.0015062	-0.013184582	0.12478736	0.005128972
2.73	0.235	0.012426601	0.0015062	-0.012376582	0.12637364	0.00555198
2.77	0.233	0.012325521	0.0015062	-0.012164582	0.12848868	0.005657732
2.8	0.237	0.012510666	0.0015062	-0.012584582	0.13007496	0.005446228
2.83	0.247	0.01294487	0.0015062	-0.013564582	0.13166124	0.004917468
2.87	0.262	0.013639696	0.0015062	-0.014847082	0.13377628	0.004124328
2.9	0.282	0.014422661	0.0015062	-0.016207082	0.13536256	0.003066808
2.93	0.306	0.015287049	0.0015062	-0.017311082	0.13694884	0.001797784
2.97	0.332	0.016157842	0.0015062	-0.017857082	0.13906388	0.000423008
3	0.359	0.016933345	0.0015062	-0.017708582	0.14065016	-0.001004644
3.03	0.383	0.017538849	0.0015062	-0.016964582	0.14223644	-0.002273668
3.07	0.406	0.017988098	0.0015062	-0.015711082	0.14435148	-0.003489816
3.1	0.426	0.018376511	0.0015062	-0.014191082	0.14593776	-0.004547336
3.13	0.437	0.01851584	0.0015062	-0.013184582	0.14752404	-0.005128972
3.17	0.445	0.01858732	0.0015062	-0.012376582	0.14963908	-0.00555198
3.2	0.448	0.018643403	0.0015062	-0.012057082	0.15122536	-0.005710608
3.23	0.446	0.018638204	0.0015062	-0.012271082	0.15281164	-0.005604856
3.27	0.437	0.0185592	0.0015062	-0.013184582	0.15492668	-0.005128972
3.3	0.423	0.018339075	0.0015062	-0.014444582	0.15651296	-0.004388708
3.33	0.404	0.017978085	0.0015062	-0.015841082	0.15809924	-0.003384064
3.37	0.382	0.017541886	0.0015062	-0.017007082	0.16021428	-0.002220792
3.4	0.355	0.016828568	0.0015062	-0.017776582	0.16180056	-0.00079314
3.43	0.329	0.016060482	0.0015062	-0.017828582	0.16338684	0.000581636
3.47	0.305	0.015222481	0.0015062	-0.017276582	0.16550188	0.00185066
3.5	0.28	0.014316336	0.0015062	-0.016089082	0.16708816	0.00317256
3.53	0.262	0.013588595	0.0015062	-0.014847082	0.16867444	0.004124328
3.57	0.247	0.012903551	0.0015062	-0.013564582	0.17078948	0.004917468
3.6	0.237	0.012485838	0.0015062	-0.012584582	0.17237576	0.005446228
3.63	0.234	0.012351174	0.0015062	-0.012271082	0.17396204	0.005604856
3.67	0.237	0.012510131	0.0015062	-0.012584582	0.17607708	0.005446228
3.7	0.248	0.012973427	0.0015062	-0.013657082	0.17766336	0.004864592
3.73	0.263	0.013601232	0.0015062	-0.014924582	0.17924964	0.004071452
3.77	0.282	0.014421749	0.0015062	-0.016207082	0.18136468	0.003066808
3.8	0.304	0.015227598	0.0015062	-0.017241082	0.18295096	0.001903536
3.83	0.331	0.016115048	0.0015062	-0.017848582	0.18453724	0.000475884
3.87	0.357	0.016874849	0.0015062	-0.017744582	0.18665228	-0.000898892
3.9	0.384	0.017568928	0.0015062	-0.016921082	0.18823856	-0.002326544
3.93	0.412	0.018228978	0.0015062	-0.015297082	0.18982484	-0.003807072
3.97	0.43	0.0184951	0.0015062	-0.013839082	0.19193988	-0.00475884
4	0.442	0.018663662	0.0015062	-0.012687082	0.19352616	-0.005393352
4.03	0.45	0.018764002	0.0015062	-0.011839082	0.19511244	-0.00581636
4.07	0.45	0.018702148	0.0015062	-0.011839082	0.19722748	-0.00581636
4.1	0.447	0.018667133	0.0015062	-0.012164582	0.19881376	-0.005657732
4.13	0.438	0.018531617	0.0015062	-0.013087082	0.20040004	-0.005181848
4.17	0.424	0.018367472	0.0015062	-0.014361082	0.20251508	-0.004441584
4.2	0.403	0.017947744	0.0015062	-0.015904582	0.20410136	-0.003331188

4.23	0.38	0.01743901	0.0015062	-0.017089082	0.20568764	-0.00211504
4.27	0.356	0.016857769	0.0015062	-0.017761082	0.20780268	-0.000846016
4.3	0.329	0.016060715	0.0015062	-0.017828582	0.20938896	0.000581636
4.33	0.304	0.015238931	0.0015062	-0.017241082	0.21097524	0.001903536
4.37	0.28	0.014317289	0.0015062	-0.016089082	0.21309028	0.00317256
4.4	0.262	0.013589594	0.0015062	-0.014847082	0.21467656	0.004124328
4.43	0.247	0.012959207	0.0015062	-0.013564582	0.21626284	0.004917468
4.47	0.238	0.012515678	0.0015062	-0.012687082	0.21837788	0.005393352
4.5	0.235	0.012380594	0.0015062	-0.012376582	0.21996416	0.00555198
4.53	0.237	0.01247834	0.0015062	-0.012584582	0.22155044	0.005446228
4.57	0.246	0.012913786	0.0015062	-0.013471082	0.22366548	0.004970344
4.6	0.262	0.013570837	0.0015062	-0.014847082	0.22525176	0.004124328
4.63	0.28	0.014299262	0.0015062	-0.016089082	0.22683804	0.00317256
4.67	0.305	0.015256346	0.0015062	-0.017276582	0.22895308	0.00185066
4.7	0.329	0.016055977	0.0015062	-0.017828582	0.23053936	0.000581636
4.73	0.357	0.016890554	0.0015062	-0.017744582	0.23212564	-0.000898892
4.77	0.384	0.017569604	0.0015062	-0.016921082	0.23424068	-0.002326544
4.8	0.41	0.018171105	0.0015062	-0.015439082	0.23582696	-0.00370132
4.83	0.433	0.018650954	0.0015062	-0.013564582	0.23741324	-0.004917468
4.87	0.446	0.018782085	0.0015062	-0.012271082	0.23952828	-0.005604856
4.9	0.452	0.018823306	0.0015062	-0.011617082	0.24111456	-0.005922112
4.93	0.455	0.018850533	0.0015062	-0.011276582	0.24270084	-0.00608074
4.97	0.453	0.018843064	0.0015062	-0.011504582	0.24481588	-0.005974988
5	0.445	0.018736618	0.0015062	-0.012376582	0.24640216	-0.00555198
5.03	0.432	0.018535368	0.0015062	-0.013657082	0.24798844	-0.004864592
5.07	0.414	0.018270202	0.0015062	-0.015151082	0.25010348	-0.003912824
5.1	0.389	0.017702904	0.0015062	-0.016688582	0.25168976	-0.002590924
5.13	0.362	0.017015784	0.0015062	-0.017647082	0.25327604	-0.001163272
5.17	0.335	0.016237346	0.0015062	-0.017876582	0.25539108	0.00026438
5.2	0.308	0.015357151	0.0015062	-0.017377082	0.25697736	0.001692032
5.23	0.285	0.014526949	0.0015062	-0.016376582	0.25856364	0.00290818
5.27	0.266	0.013708194	0.0015062	-0.015151082	0.26067868	0.003912824
5.3	0.249	0.013018871	0.0015062	-0.013748582	0.26226496	0.004811716
5.33	0.239	0.012579291	0.0015062	-0.012788582	0.26385124	0.005340476
5.37	0.235	0.012380615	0.0015062	-0.012376582	0.26596628	0.00555198
5.4	0.237	0.012477937	0.0015062	-0.012584582	0.26755256	0.005446228
5.43	0.245	0.012830368	0.0015062	-0.013376582	0.26913884	0.00502322
5.47	0.258	0.013452243	0.0015062	-0.014527082	0.27125388	0.004335832
5.5	0.276	0.014180697	0.0015062	-0.015841082	0.27284016	0.003384064
5.53	0.299	0.015032088	0.0015062	-0.017048582	0.27442644	0.002167916
5.57	0.323	0.015879304	0.0015062	-0.017744582	0.27654148	0.000898892
5.6	0.35	0.016684912	0.0015062	-0.017839082	0.27812776	-0.00052876
5.63	0.374	0.017317534	0.0015062	-0.017311082	0.27971404	-0.001797784
5.67	0.397	0.01778983	0.0015062	-0.016264582	0.28182908	-0.003013932
5.7	0.419	0.018240359	0.0015062	-0.014768582	0.28341536	-0.004177204
5.73	0.431	0.018399533	0.0015062	-0.013748582	0.28500164	-0.004811716
5.77	0.441	0.018500413	0.0015062	-0.012788582	0.28711668	-0.005340476
5.8	0.444	0.018527228	0.0015062	-0.012481082	0.28870296	-0.005499104
5.83	0.443	0.018521898	0.0015062	-0.012584582	0.29028924	-0.005446228
5.87	0.436	0.018471221	0.0015062	-0.013281082	0.29240428	-0.005076096
5.9	0.423	0.018269787	0.0015062	-0.014444582	0.29399056	-0.004388708
5.93	0.406	0.017969692	0.0015062	-0.015711082	0.29557684	-0.003489816
5.97	0.384	0.017555195	0.0015062	-0.016921082	0.29769188	-0.002326544
6	0.361	0.016986041	0.0015062	-0.017668582	0.29927816	-0.001110396
6.03	0.337	0.016306815	0.0015062	-0.017884582	0.30086444	0.000158628
6.07	0.31	0.01541657	0.0015062	-0.017439082	0.30297948	0.00158628
6.1	0.288	0.014616043	0.0015062	-0.016537082	0.30456576	0.002749552
6.13	0.268	0.013835	0.0015062	-0.015297082	0.30615204	0.003807072
6.17	0.253	0.013137335	0.0015062	-0.014104582	0.30826708	0.004600212
6.2	0.243	0.012697459	0.0015062	-0.013184582	0.30985336	0.005128972
6.23	0.237	0.012445924	0.0015062	-0.012584582	0.31143964	0.005446228
6.27	0.24	0.012565737	0.0015062	-0.012889082	0.31355468	0.0052876
6.3	0.247	0.012888417	0.0015062	-0.013564582	0.31514096	0.004917468
6.33	0.259	0.013415281	0.0015062	-0.014608582	0.31672724	0.004282956
6.37	0.277	0.014209131	0.0015062	-0.015904582	0.31884228	0.003331188
6.4	0.3	0.01506073	0.0015062	-0.017089082	0.32042856	0.00211504

6.43	0.324	0.015885433	0.0015062	-0.017761082	0.32201484	0.000846016
6.47	0.349	0.016655669	0.0015062	-0.017848582	0.32412988	-0.000475884
6.5	0.375	0.017347493	0.0015062	-0.017276582	0.32571616	-0.00185066
6.53	0.401	0.017967914	0.0015062	-0.016028582	0.32730244	-0.003225436
6.57	0.421	0.018300164	0.0015062	-0.014608582	0.32941748	-0.004282956
6.6	0.436	0.018547433	0.0015062	-0.013281082	0.33100376	-0.005076096
6.63	0.446	0.018685961	0.0015062	-0.012271082	0.33259004	-0.005604856
6.67	0.45	0.018703725	0.0015062	-0.011839082	0.33470508	-0.00581636
6.7	0.446	0.018609766	0.0015062	-0.012271082	0.33629136	-0.005604856
6.73	0.439	0.018508674	0.0015062	-0.012988582	0.33787764	-0.005234724
6.77	0.427	0.018386406	0.0015062	-0.014104582	0.33999268	-0.004600212
6.8	0.41	0.018086308	0.0015062	-0.015439082	0.34157896	-0.00370132
6.83	0.386	0.017562629	0.0015062	-0.016831082	0.34316524	-0.002432296
6.87	0.363	0.017044496	0.0015062	-0.017624582	0.34528028	-0.001216148
6.9	0.339	0.016365699	0.0015062	-0.017888582	0.34686656	5.2876E-05
6.93	0.312	0.015513142	0.0015062	-0.017497082	0.34845284	0.001480528
6.97	0.29	0.014675737	0.0015062	-0.016639082	0.35056788	0.0026438
7	0.268	0.013836004	0.0015062	-0.015297082	0.35215416	0.003807072
7.03	0.253	0.013198243	0.0015062	-0.014104582	0.35374044	0.004600212
7.07	0.242	0.01266863	0.0015062	-0.013087082	0.35585548	0.005181848
7.1	0.237	0.012446094	0.0015062	-0.012584582	0.35744176	0.005446228
7.13	0.238	0.01248455	0.0015062	-0.012687082	0.35902804	0.005393352
7.17	0.244	0.012799468	0.0015062	-0.013281082	0.36114308	0.005076096
7.2	0.256	0.01332612	0.0015062	-0.014361082	0.36272936	0.004441584
7.23	0.273	0.014024969	0.0015062	-0.015644582	0.36431564	0.003542692
7.27	0.295	0.01491297	0.0015062	-0.016876582	0.36643068	0.00237942
7.3	0.318	0.01570862	0.0015062	-0.017647082	0.36801696	0.001163272
7.33	0.345	0.016543642	0.0015062	-0.017876582	0.36960324	-0.00026438
7.37	0.37	0.017201051	0.0015062	-0.017439082	0.37171828	-0.00158628
7.4	0.395	0.017792371	0.0015062	-0.016376582	0.37330456	-0.00290818
7.43	0.418	0.018279934	0.0015062	-0.014847082	0.37489084	-0.004124328
7.47	0.434	0.018489534	0.0015062	-0.013471082	0.37700588	-0.004970344
7.5	0.447	0.018715991	0.0015062	-0.012164582	0.37859216	-0.005657732
7.53	0.452	0.018774121	0.0015062	-0.011617082	0.38017844	-0.005922112
7.57	0.452	0.018785836	0.0015062	-0.011617082	0.38229348	-0.005922112
7.6	0.446	0.018713776	0.0015062	-0.012271082	0.38387976	-0.005604856
7.63	0.435	0.018556565	0.0015062	-0.013376582	0.38546604	-0.00502322
7.67	0.419	0.018349924	0.0015062	-0.014768582	0.38758108	-0.004177204
7.7	0.395	0.017826423	0.0015062	-0.016376582	0.38916736	-0.00290818
7.73	0.37	0.017222271	0.0015062	-0.017439082	0.39075364	-0.00158628
7.77	0.345	0.01654218	0.0015062	-0.017876582	0.39286868	-0.00026438
7.8	0.319	0.015719436	0.0015062	-0.017668582	0.39445496	0.001110396
7.83	0.294	0.014851299	0.0015062	-0.016831082	0.39604124	0.002432296
7.87	0.274	0.014013409	0.0015062	-0.015711082	0.39815628	0.003489816
7.9	0.258	0.013346174	0.0015062	-0.014527082	0.39974256	0.004335832
7.93	0.244	0.012770095	0.0015062	-0.013281082	0.40132884	0.005076096
7.97	0.239	0.012505066	0.0015062	-0.012788582	0.40344388	0.005340476
8	0.239	0.012513691	0.0015062	-0.012788582	0.40503016	0.005340476
8.03	0.245	0.012781682	0.0015062	-0.013376582	0.40661644	0.00502322
8.07	0.256	0.013325159	0.0015062	-0.014361082	0.40873148	0.004441584
8.1	0.272	0.013994573	0.0015062	-0.015577082	0.41031776	0.003595568
8.13	0.294	0.014828986	0.0015062	-0.016831082	0.41190404	0.002432296
8.17	0.316	0.015649405	0.0015062	-0.017601082	0.41401908	0.001269024
8.2	0.343	0.01648485	0.0015062	-0.017884582	0.41560536	-0.000158628
8.23	0.367	0.017146249	0.0015062	-0.017524582	0.41719164	-0.001427652
8.27	0.39	0.017646227	0.0015062	-0.016639082	0.41930668	-0.0026438
8.3	0.412	0.018104532	0.0015062	-0.015297082	0.42089296	-0.003807072
8.33	0.431	0.018464354	0.0015062	-0.013748582	0.42247924	-0.004811716
8.37	0.44	0.018510823	0.0015062	-0.012889082	0.42459428	-0.0052876
8.4	0.445	0.018568565	0.0015062	-0.012376582	0.42618056	-0.00555198
8.43	0.445	0.018562556	0.0015062	-0.012376582	0.42776684	-0.00555198
8.47	0.44	0.01853668	0.0015062	-0.012889082	0.42988188	-0.0052876
8.5	0.429	0.018379228	0.0015062	-0.013928582	0.43146816	-0.004705964
8.53	0.413	0.018106089	0.0015062	-0.015224582	0.43305444	-0.003859948
8.57	0.391	0.017708015	0.0015062	-0.016588582	0.43516948	-0.002696676
8.6	0.368	0.01716299	0.0015062	-0.017497082	0.43675576	-0.001480528

8.63	0.345	0.016542626	0.0015062	-0.017876582	0.43834204	-0.00026438
8.67	0.318	0.015690529	0.0015062	-0.017647082	0.44045708	0.001163272
8.7	0.296	0.014910914	0.0015062	-0.016921082	0.44204336	0.002326544
8.73	0.274	0.014080957	0.0015062	-0.015711082	0.44362964	0.003489816
8.77	0.258	0.013347105	0.0015062	-0.014527082	0.44574468	0.004335832
8.8	0.246	0.012829582	0.0015062	-0.013471082	0.44733096	0.004970344
8.83	0.24	0.012551083	0.0015062	-0.012889082	0.44891724	0.0052876
8.87	0.239	0.012513434	0.0015062	-0.012788582	0.45103228	0.005340476
8.9	0.245	0.01278104	0.0015062	-0.013376582	0.45261856	0.00502322
8.93	0.257	0.013291208	0.0015062	-0.014444582	0.45420484	0.004388708
8.97	0.273	0.014022976	0.0015062	-0.015644582	0.45631988	0.003542692
9	0.295	0.014857537	0.0015062	-0.016876582	0.45790616	0.00237942
9.03	0.317	0.015646187	0.0015062	-0.017624582	0.45949244	0.001216148
9.07	0.344	0.016514256	0.0015062	-0.017881082	0.46160748	-0.000211504
9.1	0.371	0.017264276	0.0015062	-0.017408582	0.46319376	-0.001639156
9.13	0.397	0.017907138	0.0015062	-0.016264582	0.46478004	-0.003013932
9.17	0.42	0.018340731	0.0015062	-0.014689082	0.46689508	-0.00423008
9.2	0.436	0.018612311	0.0015062	-0.013281082	0.46848136	-0.005076096
9.23	0.447	0.018763162	0.0015062	-0.012164582	0.47006764	-0.005657732
9.27	0.451	0.018745212	0.0015062	-0.011728582	0.47218268	-0.005869236
9.3	0.451	0.018738771	0.0015062	-0.011728582	0.47376896	-0.005869236
9.33	0.443	0.018581524	0.0015062	-0.012584582	0.47535524	-0.005446228
9.37	0.433	0.018495893	0.0015062	-0.013564582	0.47747028	-0.004917468
9.4	0.415	0.018163885	0.0015062	-0.015076582	0.47905656	-0.0039657
9.43	0.395	0.017768125	0.0015062	-0.016376582	0.48064284	-0.00290818
9.47	0.371	0.017250707	0.0015062	-0.017408582	0.48275788	-0.001639156
9.5	0.347	0.016601358	0.0015062	-0.017864582	0.48434416	-0.000370132
9.53	0.321	0.015807563	0.0015062	-0.017708582	0.48593044	0.001004644
9.57	0.298	0.014970528	0.0015062	-0.017007082	0.48804548	0.002220792
9.6	0.278	0.014199543	0.0015062	-0.015967082	0.48963176	0.003278312
9.63	0.259	0.013441279	0.0015062	-0.014608582	0.49121804	0.004282956
9.67	0.247	0.012859671	0.0015062	-0.013564582	0.49333308	0.004917468
9.7	0.241	0.0125808	0.0015062	-0.012988582	0.49491936	0.005234724
9.73	0.239	0.01250096	0.0015062	-0.012788582	0.49650564	0.005340476
9.77	0.244	0.012751001	0.0015062	-0.013281082	0.49862068	0.005076096
9.8	0.254	0.013202102	0.0015062	-0.014191082	0.50020696	0.004547336
9.83	0.269	0.013837451	0.0015062	-0.015368582	0.50179324	0.003754196
9.87	0.289	0.014680287	0.0015062	-0.016588582	0.50390828	0.002696676
9.9	0.312	0.01549864	0.0015062	-0.017497082	0.50549456	0.001480528
9.93	0.338	0.016333171	0.0015062	-0.017887082	0.50708084	0.000105752
9.97	0.361	0.016970701	0.0015062	-0.017668582	0.50919588	-0.001110396
10	0.384	0.017525707	0.0015062	-0.016921082	0.51078216	-0.002326544
Average:	N/A	0.015960669	0.0015062	-0.014919486	0.24640216	-5.00653E-05

Table 6: Uncertainty at Each Data Point-Equation Rearrangement Method-Part 1

Sum of Squared Terms	**Final Uncertainty**
$T_1^2+T_2^2+T_3^2+T_4^2+T_5^2$	$\sqrt{T_1^2+T_2^2+T_3^2+T_4^2+T_5^2}$
0.000502981	0.022427248
0.000441201	0.021004795
0.000388114	0.019700613
0.000350766	0.018728753
0.000331859	0.018216984
0.000329076	0.018140439
0.000347863	0.01865108
0.000385142	0.019625041
0.000443582	0.021061392
0.000500505	0.022371978
0.000557729	0.02361628
0.000593005	0.024351684
0.000605919	0.02461542
0.000595785	0.024408697
0.000565973	0.023790188
0.000543155	0.023305694
0.000528754	0.022994656

0.00051965	0.022795843
0.00051957	0.022794081
0.000526833	0.022952846
0.000542806	0.02329819
0.000564728	0.023764006
0.000590158	0.02429316
0.000605997	0.024617002
0.000597924	0.024452494
0.000565125	0.023772353
0.000511836	0.022623795
0.000454214	0.021312289
0.000400813	0.020020303
0.000353542	0.018802723
0.000331866	0.018217185
0.000331454	0.018205877
0.000347846	0.018650638
0.000382355	0.0195539
0.000431154	0.020764247
0.000495899	0.022268784
0.000549235	0.023435768
0.000586945	0.024226938
0.000604535	0.024587297
0.000599853	0.024491901
0.000582079	0.024126321
0.000557801	0.023617812
0.000540487	0.023248369
0.000529906	0.023019684
0.000528162	0.022981772
0.000535772	0.023146754
0.000550606	0.023464988
0.000573984	0.023957962
0.000593971	0.024371529
0.000604693	0.024590514
0.000596392	0.024421146
0.000566646	0.023804335
0.000517699	0.022752998
0.000456796	0.021372794
0.000403576	0.020089208
0.000360315	0.018981967
0.000337334	0.018366662
0.000334177	0.018280517
0.000346832	0.018623424
0.000385094	0.019623822
0.000433781	0.020827421
0.000491848	0.022177637
0.000549219	0.023435418
0.000590046	0.024290855
0.000605685	0.024610672
0.000597261	0.024438915
0.000573619	0.023950339
0.000550226	0.023456904
0.000533193	0.023090965
0.000522896	0.02286691
0.000520039	0.022804359
0.000526788	0.022951862
0.000538631	0.023208428
0.000559486	0.02365345
0.000589477	0.024279152
0.000604561	0.024587829
0.000599728	0.024489351
0.000573794	0.023953993
0.000523985	0.022890723
0.000468734	0.021650263
0.000406333	0.020157714
0.000365869	0.019127699
0.000340693	0.018457879

0.000334174	0.01828043
0.000346818	0.018623064
0.000378018	0.019442677
0.000425756	0.020633853
0.000482357	0.021962619
0.000538868	0.023213532
0.000582399	0.024132941
0.00060361	0.024568476
0.000602846	0.024552932
0.000584857	0.024183821
0.00056203	0.023707169
0.000545245	0.023350471
0.000531761	0.023059951
0.000527829	0.022974537
0.000531645	0.023057431
0.000546852	0.023384869
0.000566497	0.023801198
0.000587872	0.024246071
0.000604159	0.024579649
0.000602105	0.024537834
0.000578404	0.024050039
0.000535898	0.023149467
0.00047615	0.021820857
0.000424364	0.020600109
0.00037695	0.019415191
0.000346198	0.018606394
0.000336814	0.018352493
0.000346805	0.018622704
0.000380759	0.019513037
0.000426582	0.020653862
0.00048233	0.021962019
0.000535027	0.023130645
0.000580762	0.024098999
0.000602707	0.024550099
0.000602672	0.024549372
0.000583059	0.02414661
0.000558504	0.023632692
0.000540651	0.023251908
0.00052835	0.022985872
0.000526033	0.022935407
0.000530717	0.023037306
0.000543813	0.023319793
0.000565601	0.023782368
0.000588443	0.024257838
0.000602898	0.024553978
0.000602625	0.024548417
0.000578412	0.024050194
0.000535372	0.023138107
0.000476177	0.021821482
0.000424392	0.020600768
0.000378389	0.019452225
0.000348961	0.018680501
0.000339552	0.018426937
0.000346011	0.018601363
0.000375209	0.019370309
0.000423882	0.020588399
0.000475661	0.021809659
0.00053693	0.023171749
0.00057826	0.024047031
0.000603238	0.024560897
0.000602695	0.024549857
0.000584523	0.024176904
0.000558306	0.023628502
0.000537029	0.023173891
0.000526614	0.022948061
0.000521748	0.022841802

0.000525386	0.022921291
0.000537334	0.023180461
0.000556009	0.023579836
0.000580934	0.024102581
0.000600883	0.024512917
0.000604578	0.024588173
0.000585562	0.024198391
0.000542937	0.023301002
0.000489951	0.022134833
0.000435049	0.020857821
0.000383936	0.019594279
0.000352576	0.018776999
0.000339553	0.018426951
0.000346001	0.018601093
0.000371053	0.019262727
0.000413067	0.02032405
0.000465753	0.021581301
0.000523586	0.022882009
0.000570099	0.023876749
0.000599167	0.024477896
0.000605071	0.024598195
0.000592367	0.024338593
0.000570539	0.023885966
0.000552988	0.023515688
0.000536602	0.023164681
0.000531544	0.023055246
0.000533362	0.023094641
0.000545609	0.023358265
0.00056396	0.023747852
0.000584195	0.024170134
0.000602189	0.024539547
0.000604206	0.024580602
0.000588064	0.024250037
0.000546577	0.023378989
0.000496932	0.022291982
0.00044217	0.021027848
0.000394959	0.019873585
0.000363634	0.019069181
0.000345203	0.018579633
0.000354254	0.018821624
0.000376559	0.019405136
0.000413993	0.020346813
0.000468221	0.021638406
0.000525604	0.022926063
0.000570787	0.023891158
0.000598478	0.024463816
0.000605109	0.024598971
0.000592433	0.024339956
0.000568919	0.023852023
0.00054843	0.023418578
0.000533428	0.023096053
0.000526092	0.022936693
0.000530586	0.02303445
0.000540945	0.02325823
0.00056043	0.023673398
0.000581448	0.024113237
0.000599916	0.024493182
0.000604888	0.024594479
0.000590109	0.024292157
0.000551266	0.023479056
0.000501495	0.022394075
0.000442198	0.021028509
0.000396563	0.019913901
0.000360886	0.018997002
0.000345207	0.018579747
0.000348183	0.018659661

0.000368249	0.019189812
0.000405822	0.020145035
0.000456272	0.021360525
0.000515146	0.022696828
0.000561802	0.023702365
0.000595603	0.024404975
0.000604783	0.024592329
0.000595487	0.024402603
0.000573871	0.023955595
0.000550306	0.023458599
0.000532544	0.023076914
0.000524764	0.022907733
0.000525204	0.022917336
0.000534468	0.023118562
0.00055078	0.023468712
0.000574548	0.02396974
0.0005967	0.024427442
0.000605513	0.024607177
0.000595554	0.024403984
0.000562781	0.023723008
0.000512031	0.022628104
0.000457661	0.021393017
0.000410225	0.020254001
0.000367498	0.019170234
0.000350714	0.018727355
0.00035093	0.018733115
0.000369806	0.019230333
0.000405797	0.0201444
0.00045369	0.021300007
0.000511369	0.022613466
0.000558581	0.023634319
0.000593902	0.024370112
0.000605412	0.024605114
0.000597507	0.02444395
0.000578537	0.024052801
0.000555377	0.02356644
0.000539006	0.02321651
0.000531065	0.023044837
0.000530841	0.023039996
0.000539964	0.023237132
0.000556216	0.023584236
0.000576786	0.024016374
0.000598296	0.024460081
0.000605177	0.02460034
0.000595569	0.024404286
0.000561234	0.023690379
0.00051634	0.022723112
0.000459559	0.021437325
0.000410249	0.020254615
0.000373041	0.019314274
0.000353885	0.018811844
0.000350923	0.018732944
0.000369789	0.019229907
0.000406832	0.020170066
0.000456216	0.021359216
0.000513496	0.022660444
0.000559177	0.023646918
0.000594767	0.024387847
0.000606069	0.024618477
0.000596555	0.024424469
0.000572314	0.02392308
0.000550841	0.023469994
0.000534312	0.023115187
0.000525659	0.022927258
0.000525418	0.022921993
0.000535575	0.023142488

0.000552546	0.023506298
0.000575225	0.023983859
0.000594625	0.024384931
0.000605601	0.024608964
0.000597154	0.024436735
0.000566751	0.02380653
0.000520558	0.022815742
0.000469591	0.021670042
0.000414691	0.020363963
0.000375819	0.019386056
0.000356651	0.0188852
0.000350611	0.018724614
0.000367011	0.019157519
0.000398629	0.019965701
0.000444031	0.021072043
0.000500233	0.02236588
0.000550816	0.023469477
0.000589	0.024269322
0.000603685	0.024570004
0.000601155	0.02451846
0.000505923	0.022400149

Table 7: Uncertainty at Each Data Point-Equation Rearrangement Method-Part 2

16.6 Appendix F: Logbook

Figure 16: Pictures of Logbook

Oscillation Lab

2015-11-18 Purpose: The purpose of this lab is to determine the spring constant, k. Furthermore, the second portion of this lab will aim to confirm the validity of the law of conservation of energy.

Part 1. Determining the spring constant.
Method: Use formula:
$\Sigma F = ma$
$F_s = F_g$

Trial #	Mass (g) (not including stand)	Bottom Position (m) x_f	Top Position (m) x_i	Length of Spring (m)	Force Exerted by Spring (N)
1	150 g	51 cm	57 cm	0.184 − x_f	
2	250 g	49 cm	57 cm		
3	350 g	47 cm	57 cm		
4	450 g	44 cm	57 cm		
5	550 g	37 cm	57 cm		
6	650 g	31 cm	57 cm		
7	750 g	27.5 cm	57 cm		
8	850 g	25.5 cm	57 cm		
9	950 g	22.5	57 cm		
10	450 g				
11	400 g	40.5 cm			
12	100 g	53 cm			
13	200 g				
14	300 g	50 cm			
15	400 g	46 cm			
16	500 g	42 cm			
17	600 g	38.5 cm			
18	700 g	35.5 cm			
19	800 g	31.0 cm			
20	900 g	27.5 cm			
21	75 g	22.5 cm			
22	175 g	54.0 cm			
23	275 g	50.5 cm			
24	375 g	47.0 cm			
		43.0 cm			

Stand Mass: add to each mass

To calculate F_s: $F_s = F_g$
$F_s = mg$ ← only m and g are required
However, the stretch length is also needed in order to determine the spring constant, k.
Stretch Length $= S_f - S_{relaxed}$
$F_s = F_g$, $kx = mg$, $k = \frac{mg}{x}$, $x = x_f - x_i$ (position final, initial)
$k = \frac{mg}{x_f - x_i}$

Part 2: Law of Conservation of Energy
Measurements in Excel, as machine measures st.
Formula: $E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + mgh$
$x_{eq} = \left(\frac{-mg}{k}\right)$, $h = (h_{eq} + y)$
$E_T = \frac{1}{2}mv^2 + \frac{1}{2}ky^2 + mgh_{eq} + \frac{(mg)^2}{2k}$
need to determine v and y
2.8 grams
$(400 + 90 + 3.2?)$ g
$(400 + 40 + 5.5)$ g
492.4 g (updated)

Spring Mass:
"100 g" =
"50 g" =
"250 g" =
"500 g" =
"25 g" =
"10 g" =
"10 g" =
"5 g" =
"20 g" =
All MASSES EQUAL To supposed mass
taken into account in uncertainty

For other page.

Trial #	Mass (g)	x_f
26	575	37 cm
27	675	33 cm
28	775	29.5 cm
29	875	26 cm
30	975	14 cm

Part 1:
Method 1: Hooke's Law
$F_s = F_g$
$kx = mg$
$k = \frac{mg}{x}$
Method 2: Line of Best / Scatterplot
mg vs. Stretch Length (m)
X-int: initial equilibrium (relaxed spring)
Slope: K spring constant

Part 2: Law of Conservation of Energy
Kinetic Energy: $\frac{1}{2}mv^2$
Gravitational Potential Energy: mgh
Spring Potential Energy: $\frac{1}{2}kx^2$
Energy summation
$\frac{1}{2}mv^2$ → given
$\frac{1}{2}kx^2 \rightarrow \frac{1}{2}k\left(\frac{-mg}{k} + h_{eq}\right)^2$
$mgh \rightarrow mgh$
Given: Mass (= ____)
Velocity (from machine)
H_{eq} (Measured = 0.34 m)
H (Position from machine)
$Y = H_{eq} - H$
$x = Y$
How to factor in mass of spring (both Parts):
$\frac{1}{3}$ of mass of spring contributes to oscillation
Other way (Equation Rearrangement)
$E_T = \frac{1}{2}mv_i^2 + \frac{1}{2}k(h - h_{eq})^2 + mgh$
$E_T = \frac{1}{2}mv^2 + \frac{1}{2}k(h - h_{eq})^2 + mgh_{eq} + \frac{(mg)^2}{2k}$
Note: $h - h_{eq} = -(h_{eq} - h)$
Need sinusoidal regression's derivative to determine v_i.