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LaTeX Homework
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Problem 1

The reduced cubic equation $y^3 + 3py + 2q = 0$ has one real and two complex solutions when $D = q^2p^3 > 0$. These are given by Cardanos formula as

$$y_1 = u + v, \quad y_2 = -\frac{u+v}{2} + \frac{i}{2}\sqrt{3}(u-v), \quad y_3 = -\frac{u+v}{2} - \frac{i}{2}\sqrt{3}(u-v)$$

where

$$u = \sqrt[3]{-q + \sqrt{q^2 + p^3}}, \quad v = \sqrt[3]{-q - \sqrt{q^2 + p^3}}.$$

Problem 2

Each of the measurements $x_1 < x_2 \cdots < x_r$, occurs $p_1, p_2, ..., p_r$ times. The mean value and standard deviation are then

$$x = \frac{1}{n}\sum_{i=1}^{r} p_i x_i, \quad s = \sqrt{\frac{1}{n}\sum_{i=1}^{r} p_i (x_1 - x)^2}$$

where $n = p_1 + p_2 + \cdots + p_r$.

Problem 3

Although this equation looks very complicated, it should not present any great difficulties:

$$\int \frac{\sqrt{(ax+b)^3}}{x}\,dx = \frac{\sqrt[2]{(ax+b)^3}}{3} + 2b\sqrt{ax+b} + b^2 \int \frac{dx}{x\sqrt{ax+b}}.$$

The same applies to $\int_1^8 \left(\frac{dx}{\sqrt[3]{x}}\right) = \frac{3}{2}(8^{2/3} + 1^{2/3}) = \frac{15}{2}$.

Problem 4

The gamma function $\Gamma(x)$ is defined as

$$\Gamma(x) = \lim_{n\to\infty} \prod_{i=1}^{n-1} \frac{n!n^{x-1}}{x+v} \equiv \int_0^\infty e^{-t}t^{x-1}dt.$$

The integral definition is valid only for $x > 0$ (2^{nd} Euler integral).

Problem 5

The total number of permutations of n elements taken m at a time (symbol p_n^m) is

$$p_n^m = \prod_{i=0}^{m-1}(n-i) = \underbrace{n(n-1)(n-2)\cdots(n-m+1)}_{\text{total of m factors}} = \frac{n!}{(n-m)!}.$$

Problem 6

$$\mathcal{M}^{\beta}(\tau_0,\psi_0) = \frac{2\mathcal{C}_4\tilde{\beta}}{3\mathcal{C}_1} + \frac{2\pi\mathcal{C}_3\tilde{\eta}\tilde{\omega}^2}{\mathcal{C}_1^2}\left[\frac{\sin(\tilde{\omega}\tau_0+\psi_0)}{\sinh[\pi\tilde{\omega}(2C_1)]} + \tilde{\eta}\frac{\sin(2\tilde{\omega}\tau_0+\psi_0)}{\sinh[\pi\tilde{\omega}(C_1)]}\right].$$

Problem 7

$$\sum\nolimits^{\psi_0} = \{(\theta,\varphi,\psi)\in\mathbb{R}\times\mathbb{R}\times S^1|\psi=\psi_0\}.$$

Problem 8

For integers m,n with $n \geq 4$ even, and $2 \leq m < n$, the expansion factor of $SK_{m,n}$ is given by

$$\varepsilon(SK_{m,n}) = \begin{cases} \frac{n}{2m} & \text{if } 2 \leq m < \frac{n}{4} \\ 2 & \text{if } \frac{n}{4} \leq m < \frac{n}{2} \\ \frac{3n-2-2m}{n} & \text{if } \frac{n}{2} \leq m < \frac{3n}{4} \\ 1+\frac{2}{n}\lfloor\frac{n-2}{4}\rfloor & \text{if } \frac{3n}{4} \leq m < n. \end{cases}$$