Consider the graph of $y = 2t - 2$.

(1) Sketch the region below this line, above the t-axis, and between the vertical lines $t = 1$ and $t = 4$.
(2) Use geometry to find the area of the region.
(3) Now sketch the region below the line $y = 2t - 2$, above the t-axis, and between the lines $t = 1$ and $t = x$ for some $x > 1$.
(4) Use geometry to find the area of this region as a function of x. Call this area, your function, $A(x)$.
(5) Take the derivative of the area function $A(x)$.

Now, consider the function $y = 3 + t^2$.

(1) For some $x > 1$, sketch the region that the function $A(x) = \int_{-1}^{x} (3 + t^2)\, dt$ represents the area of.
(2) Use the fact that $\int_a^b u^2\, du = \frac{b^3 - a^3}{3}$ and $\int_a^b c\, du = c(b - a)$, and the rules of combining definite integrals to find an expression for $A(x)$ and simplify that expression.
(3) Compute $A'(x)$.
(4) For a small positive number h, sketch the region whose area is represented by $A(x + h) - A(x)$.
(5) Use your picture, and maybe a rectangle, to explain why $\frac{A(x + h) - A(x)}{h} \approx 3 + x^2$.
(6) Based on part (e), give both an intuitive reason and a logical reason using the limit definition of the derivative for why your answer in (c) makes sense.

Suppose that f is a continuous function. Define a new function g by $g(x) = \int_a^x f(t)\, dt$, where a is a real number and $x > a$. Based on your above work take a guess at what $g'(x)$ is.

THIS IS PART ONE OF THE FUNDAMENTAL THEOREM OF CALCULUS!

Now you've seen that for a continuous function f, that if $\int_a^x f(t)\,dt$, then $g'(x) = f(x)$. Remember that the lower limit, a, is a number, while the upper limit, x, is the variable which we are taking the derivative with respect to. Use this theorem to find $g'(x)$ in the following. Practice applying this new differentiation rule and combining it with previous rules as well as the properties of definite integrals that we learned in section 5.2

(1) $g(x) = \int_\pi^x \sin(2t)\,dt.$

(2) $g(x) = \int_x^7 t^3 - \frac{1}{t}\,dt.$

(3) $g(x) = \int_0^{x^4} \sec t\,dt.$

(4) $g(x) = \int_{2t}^{3t} \frac{3t+1}{t^2+1}\,dt.$

Let $g(x) = \int_a^x f(t)\,dt$, for a continuous function f on the interval $[a, b]$. Let's also suppose that $F(x) = \int f(x)\,dx$.

(1) What are $g'(x)$ and $F'(x)$? What does that tell you about the difference (like subtraction) between $g(x)$ and $F(x)$?
(2) Write an equation that shows what you concluded in part (a).
(3) Compute $g(a)$ in two ways: using the definition of g, and also using your formula from part (b).
(4) These two methods should give you the same solution, setting these two answers equal allows you to solve for a constant. Do it.
(5) Rewrite the definite integral $\int_a^b f(t)\,dt$ in terms of the function g.
(6) Use the formula, finished in part (d) when you solved for the constant, to solve the definite integral in terms of F.

THIS IS PART TWO OF THE FUNDAMENTAL THEOREM OF CALCULUS!

This is the portion of the FUNdamental Theorem that we will use most often. Now, go forth and integrate!

(1) $\int_0^1 x^{4/5}\,dx$

(2) $\int_1^e \frac{1}{x}\,dx$

(3) $\int_{-1}^1 e^{u+2}\,du$

(4) $\int_{-\pi/2}^{\pi/2} \cos t\,dt$