Density Of Ice

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The density of solid water, unlike most molecules, is less than that of its liquid form. Its precise value is of use in many applications, including . . .. Freezing a spherical droplet of water and analyzing the changed shape from a sphere to a sphere with a slight peak in order to find the density of ice. We find the density of ice to be at $0.90 \pm 1.66 \cdot 10^6$ g/mL. The precision of our measurement was limited by uncertainty in the angle measurements of the peak of the droplet.

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Ice floating on water is such a common occurrence in everyday life. Typically, substances have liquid forms that are less dense than their solid form. The fascinating, counter-intuitive consequences of ice's lighter density has biological applications, such as forming ice covers to protect aquatic life in frozen lakes over the winter [1]. In addition to satisfying curiosity, precise knowledge of the density of ice is necessary in determining whether or not a frozen lake can sustain life beneath it, or keep life afloat above it... [1].

Water droplets are held in a spherical shape by its property of cohesion in the hydrogen bonds of water. As it freezes, the water becomes less dense, and expands. Due to the force of gravity, the spherical droplet takes on a slight peak once frozen solid [2] The value obtained, $0.90 \pm 1.66 \cdot 10^6$ g/mL, was limited in accuracy by the pictures taken of the droplet and the accuracy of the image processing program, ImageJ [1]. The expected value of the density of ice was 0.917 g/mL [1].

Here we report a simple approach to measuring the density of ice using volume ratio analysis of water droplets and ice peaks $0.90 \pm 1.66 \cdot 10^6$ g/mL. The precision of this measurement was limited by uncertainty in the angle measurements of the peak of the droplets.

Our approach was based on the one described by J. Watz *et al.* [1]. We placed water droplets using a hospital-grade micro pipette on a metal sheet cooled by dry ice to a temperature of -78.5'C. The droplet would freeze from the bottom upwards, forming the distinct peak-like shape. It would take around ten to thirty seconds to freeze.

Once completely frozen, we immediately took a picture. If the picture was not captured immediately, sometimes the peak would collapse on itself, rendering that trial invalid for analysis.

To find the density of ice, we followed the method outlined by Schetnikov, Matiunin, and Chernov [3]. They found that the density ratio of ice to water, ρ was only dependent on the vertex angle α.They found the mathematical relationship by removing the top of the water droplet before it froze entirely, leaving a dip (Fig. 1). By comparing that to the droplet allowed to freeze entirely, they found the following relationships to be true:

$$V_1 = \frac{\pi r^3}{3} \frac{(1 - cos\alpha)^2 (2 + cos\alpha)}{sin^3\alpha}$$

$$V_2 = \frac{\pi r^3}{3} \frac{(1 - sin\alpha)^2 (2 + sin\alpha)}{cos^3\alpha}$$

$$V_c = \frac{\pi r^3}{3} \frac{cos\alpha}{sin\alpha}$$

$$\rho = \frac{V_1(\alpha) + V_2(\alpha)}{V_1(\alpha) + V_c(\alpha)}$$

Notice that the density ratio only depends on α. As a result, we used a USB microscope camera mounted at equal elevation to the water droplets in order to find the angle α (Fig. 2). These pictures were analyzed in ImageJ to find α. Since the density of water is 1 g/mL, we could take ρ to be equal to the density of ice in g/mL.

The measurement was repeated seventeen times. The first three trials and the last ten trials were taken in an uncovered apparatus. We initially thought fluctuations in temperature would affect the freezing of the droplet, and attempted to cover the apparatus for trials four to nine. Instead, we found that carbon dioxide gas released by the dry ice was dissolving in the droplet as it froze, causing the measured density to be lighter than expected. This problem was less noticeable when we froze the droplets in an uncovered apparatus, but may have affected our results.

We measured the vertex angle α for each of these trials. For the first few trials, we also measured the radius r, and the total volume of the droplet V_{tot}. Upon further investigation, we realized that the volume ratio ρ was only dependent on α Uncertainty in the vertex angle was due to the display of ImageJ, which only showed three

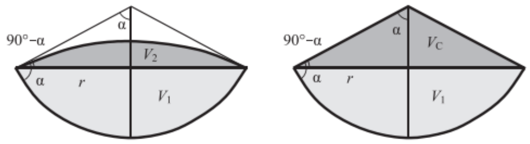

FIG. 1: Mathmatical visual of the top of the droplet and peak. On the left is the top of the spherical, unfrozen water droplet. V_2 is the volume of water taken from the drop before it froze. On the right is the peak of the frozen droplet. Notice the vertex angle, α that the density ratio ρ depends on.

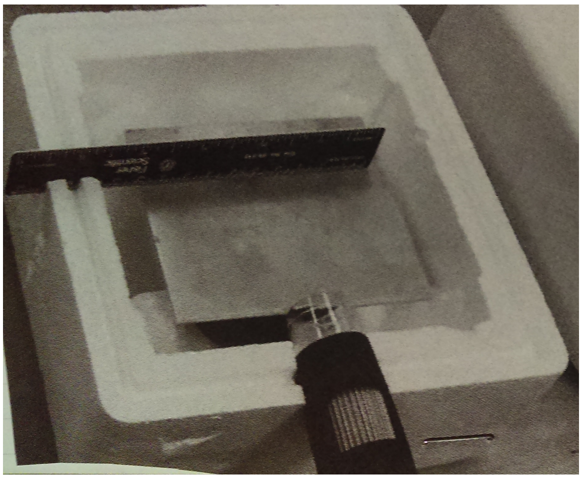

FIG. 2: Apparatus used to freeze water A properly formatted figure would have axis labels and error bars, of course, and a proper functional fit, instead of connected dots, if appropriate.

decimal places. Uncertainty was then calculated to be $\delta\alpha = \frac{0.001}{\sqrt{12}}$. Using the equations given by Schetnikov, Matiunin, and Chernov; we calculated ρ as given by each α. This data was used to calculate the uncertainty in ρ. Since ρ is a function of α,

$$\delta\rho = \frac{d\rho}{d\alpha}\delta\alpha$$

A linear fit to (Fig. 3) shows that $\frac{d\rho}{d\alpha}$ is the slope, 0.0057575 g/mL*degree.

This result is consistent with measurements reported by Schetnikov *et al.* [3], who froze water droplets inside of a cooling chamber at -15'C, likely similar to a refrigerator. It is also consistant with the accepted value of the density of ice, 0.9167 g/mL. The approach used here is less accurate than Schetnikov *et al.*, due to the issues with sublimation as mentioned previously.

Air bubbles within the water droplet was likely the cause of discrepancy between our measurement and the accepted value. We used water degassed by boiling, but perhaps future measurements could use water degassed

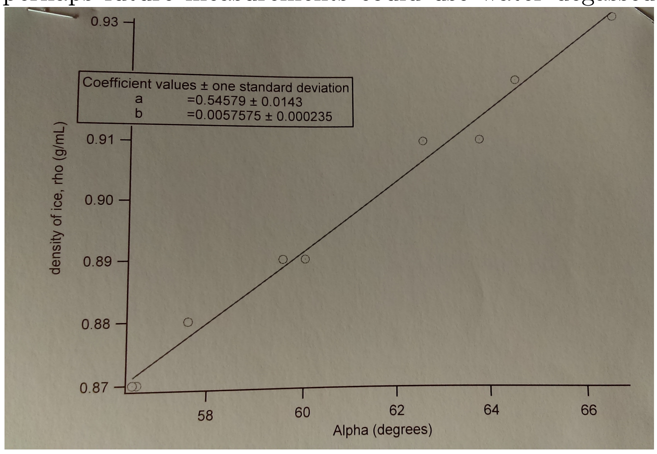

FIG. 3: The graph of the density ρ and vertex angle, α. Ideally, the slope of this graph would be zero, since there is no mathematical dependence of the density on angle. However, due to slight variations in droplets, the slope of this graph is part of the calculation to find the uncertainty in ρ.

by a vacuum. Furthermore, we believe that carbon dioxide gas from the dry ice was dissolving into the droplets before or as they froze. A full cooling chamber, such as the one used by Schetnikov *et al.*, would give a more accurate measurement.

In summary, we have measured the density of ice to be $0.90 \pm 1.66 \cdot 10^6$ g/mL. Our measurement suffered systematic errors due to the method of cooling water droplets. The most marked improvements to this measurement approach would come from purifying the water further. A better method of degassing the water and of cooling the droplet into ice would give a more accurate measurement.

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[2] A. Sanz *et al.*, "The influence of gravity on the solidification of a drop", *Journal of Crystal Growth* **82**, 81-88 (1986).

[3] A. Schetnikov *et al.*, "Conical shape of frozen water droplets", *American Journal of Physics* **83**, 36 (2010).