Analysis of Algorithms Problem Set 1

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1 Dynamic Array

Using the potential method, prove that insertion into the dynamic array has amortized cost of $O(1)$

Define a potential function

$$\Phi_i = 2i - m \tag{1}$$

where i is the ith operation of insert, and m is the size of the dynamic array in the ith operation. Assume we start with size 1.

Φ_1	$2 \times 1 - 1 = 1$
Φ_2	$2 \times 2 - 2 = 2$
Φ_3	$2 \times 3 - 4 = 2$
Φ_4	$2 \times 4 - 4 = 4$
Φ_5	$2 \times 5 - 8 = 2$
Φ_6	$2 \times 6 - 8 = 4$
...	...

Therefore, we have two condition, $when \quad i = 2^k + 1, m = 2k,\ else, m \leq 2i, k = z^*.$ when $i = 2^k + 1, m = 2k, \Phi_i = 2 \cdot 2^k + 1 - 2k > 0$

Known the Amortized Cost equation of potential method

$$\hat{c_i} = c_i + \Phi_i - \Phi_{i-1} \tag{2}$$

where c_i is the count of the ith insert

c_1	$0 + 1$
c_2	$1 + 1$
c_3	$2 + 1$
c_4	$0 + 1$
c_5	$4 + 1$
c_6	$0 + 1$
...	...

From the table 2, the c_i has 2 forms, the normal cost is $c_i = 1$, the expensive cost is $c_i = i - 1 + 1$,$i = 2^k + 1$.
In the normal case:

$$\hat{c_i} = c_i + \Phi_i - \Phi_{i-1} \quad (3)$$

$$\hat{c_i} = 1 + (2i - m) - (2(i - 1) - m) \quad (4)$$

$$\hat{c_i} = 3 \quad (5)$$