MAT 3530 Homework #1

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Question 1.1. *The* power set *of a finite set A is the set $\mathcal{P}(A)$ of all subsets of A. Determine a formula for $\#\mathcal{P}(A)$ in terms of $\#A$. Prove your formula is correct.*

Proof. If the power set of $\mathcal{P}$(A) contains the totality of all elements in A ie a∈A and A∈P(A), then there also exists a relation between the cardinality of P and the cardinality of A. We can show this by example A={1,2} therefore P(A)=[{0},{1}] ∪ P(2)=[{2}]. So the cardinality of $P(1)$ equals the cardinality of A, therefore we can show the cardinality of a added to the cardinality of $P(1)$ equals the cardinality of P(A). □

Question 1.2. *Let $f : X \to Y$ be a function, and let $A, B \subset X$.*

(i) *Prove $f(A \cup B) = f(A) \cup f(B)$.*
We first need to show f(A∪B) ⊂ f(A) ∪f(B) Let's say y∈f(A∪B), and there is x∈f(A) or x∈B so f(x)∈f(B) in either case y=f(x)∈f(A)∪f(B) secondly we need to show f(A)∪f(B) ⊂ f(A∪B) y∈f(A)∪f(B), so y∈f(A) or y∈f(B) if y∈f(A), then there is a∈A so f(a)=y such that y=f(A)∈f(A∪B) since a∈f(A∪B) also if y=f(B), then y∈f(A∪B) and the equality is complete.

(ii) *Prove $f(A \cap B) \subset f(A) \cap f(B)$. Say y∈f(A∩B) and x∈A∩B so I can then say that x∈A, y∈f(A) and x∈b and y∈f(B). So y∈⊂f(A)∩f(B).*

(iii) *Find a counterexample to $f(A \cap B) = f(A) \cap f(B)$.*

Proof. Let's say A = {1,2} and b = {1,3}. Now we multiply A, which yields A*A={1,2,2,4} and B*A={1,3,3,9}. Now, f(A∩B)={1,2} but f(A)∩f(B)={1,0}. □

Question 1.3. *Define the relation $\sim$ on $\mathbb{Q}$ by $r \sim s$ if and only if $r - s \in \mathbb{Z}$. Prove $\sim$ is an equivalence relation.*

Proof. We know that $\mathbb{Z}\subset\mathbb{Q}$ and if c∈r-s then c∈$\mathbb{Q}$ and c∈$\mathbb{Z}$ so r∼s, r∼c, and s∼c so it is transitive. also, r-r∈ $\mathbb{Z}, \mathbb{Q}$ and s-s∈ $\mathbb{Z}, \mathbb{Q}$ so it is reflexive. Finally since r-s∈ $\mathbb{Z}$ then s-r∈ $\mathbb{Z}$ because r∼s so it is symmetric. □

Question 1.4. *Let $f : X \to Y$ be a function. Prove that the relation $u \sim v$ if and only if $f(u) = f(v)$ is an equivalence relation.*

Proof. Say u∈x and v∈y if f(u)=f(v) and v∈f(v) and u∈f(u) then f(v)∼f(u) and this show reflexive as well as symmetric. If c∈y then c∈f(v),f(u) thus c∼u,c∼v, and of course u∼v which is transitive. □